Calculations Policy

Calculation Policy

For the 2014 National Curriculum

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Always think:
  Can I do it mentally?
  Can I do it with a jotting?
  Do I need a written method (vertical layout)?
  Do I need a calculator?

Progression towards a standard written method of calculation

This calculation policy has been written to be in line with the 2014 National Curriculum. It provides guidance on appropriate calculation methods and progression. The content is set out under the following headings: addition, subtraction, multiplication and division. Children should use mental methods as their first port of call when appropriate but, for calculations that they cannot do in their heads, they will need to use an efficient written method accurately and with confidence.

Aims of the policy

- To ensure consistency and progression in our approach to calculation
- To ensure that children develop an efficient, reliable, formal written method of calculation for all operations
- To ensure that children can use these methods accurately with confidence and understanding.
- To be used in conjunction with the White Rose Maths Hub planning blocks to aid the progress of the children as mathematicians.

How to use this policy

- Use the policy as the basis of your planning but ensure you use previous or following years' guidance to allow for personalised learning
- Always use assessment for learning (targets and AWOL grids) to identify suitable next steps in calculation for groups of children
- If, at any time, children are making any significant errors, return to the previous stage in the calculation
- Cross reference with other mathematic policies for guidance on key facts, key vocabulary and mental methods
- Always use suitable methods, models and images to support the children's understanding of calculation and place value as appropriate.
- Encourage children to make sensible choices about the methods they use when solving problems!
- All staff and adults teaching in the room should be following the policy (TAs, supply teachers) so ensure planning is explicit

ADDITION GUIDELINES

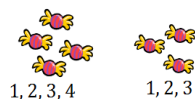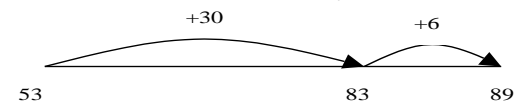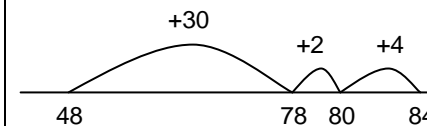| Stage One	Stage Two	Stage Three
Prerequisite skills (based on the practical) Counting numbers to 20 (using familiar / practical resources) Place numbers to 20 in order Bonds up to 10 and to make 10 1 more than a number Addition as combining groups 1, 2, 3, 4 1, 2, 3 1, 2, 3, 4, 5, 6, 7 Addition as counting on Doubling numbers within 20	**Prerequisite skills** (based on the practical) Relate number bonds to 10 to add multiples of 10 up to a total of 100 e.g. if 3 + 4 is 7 then 30 + 40 is 70 Use familiar objects to recognise the place value of 2 digit numbers. **2 4** Recognise and explain 24 is '2 tens and 4 ones' **2 0** > **4** Progressing to: **PARTITIONING AND RECOMBINING** **Partition into tens and ones and recombine** **Pre J10** (before jumping in 10s) 12 + 23 = 10 + 2 + 20 + 3 = 30 + 5 = 35 *Model this on a bead bar and practise on 100-beadstrings, showing the 'collection' of 10s and then the ones. i.e. "2 tens and 1 ten makes 3 tens, which is 30. Then 3 and 2 makes 5 ones. Altogether we can see 3 tens and 5 ones, which is 35." Check by counting in tens and ones along the bead bar. Model and practise with place value arrow cards, numicon, bead strings or Dienes, using known facts and place value to calculate each step.* **Count on in tens and ones** **J10** (jumping in 10s)	**Partition into tens and ones** • Partition one number and recombine. • Count on by partitioning the second number only e.g. 36 + 53 = 53 + 30 + 6 = 83 + 6 = 89 As modelled below as necessary +30 +6 53 83 89 Children need to be secure adding multiples of 10 to any two-digit number including those that are not multiples of 10. 48 + 36 = 84 +30 +2 +4 48 78 80 84 First J10 then T10 **Add a near multiple of 10 to a two-digit number (Overjumping - OJ)** Secure mental methods by using a number line to model the method. Continue as in Stage 2 but with appropriate numbers E.g. 35 + 19 is the same as 35 + 20 – 1. *Once a child is able to add 3 digit numbers on a number line securely move on to vertical expansion.* **+ / = signs and missing numbers** Continue using a range of equations as in Stage 1 and 2 but with appropriate, larger numbers.

ADDITION GUIDELINES

Stage One	Stage Two	Stage Three

Stage One

Number bonds to 20

+ / = signs and missing numbers

Children need to understand the concept of equality before using the '=' sign. Calculations should be written either side of the equality sign so that the sign is not just interpreted as 'the answer'.

$2 = 1 + 1$
$2 + 3 = 4 + 1$
$3 = 3$
$2 + 2 + 2 = 4 + 2$

Missing numbers need to be placed in all possible places.

$3 + 4 = \square$ $\square = 3 + 4$
$3 + \square = 7$ $7 = \square + 4$
$\square + 4 = 7$ $7 = 3 + \square$
$\square + \nabla = 7$ $7 = \square + \nabla$

The Number Line

Children use a numbered line to count on in ones. Children use number lines and practical resources to support calculation and teachers *model* the use of the number line.

e.g. 7+ 4:
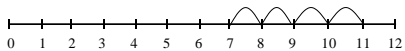
Number line Teaching Points:
Always work with numbers reading from left to right (smallest to largest), whatever the operation of the calculation.
Numbers ('landmarks') are written below the line.
Size of the 'jumps' are written above the 'jumps'.

Stage Two

$23 + 12 = 23 + 10 + 2$
              $= 33 + 2$
              $= 35$

Model this on a number line starting at 23 and jumping 10 (J10) to make 33 and then add 2 in one jump.
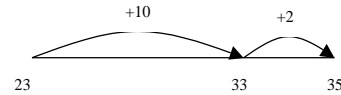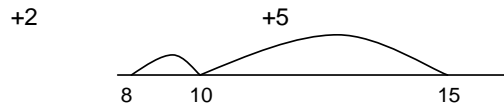

The Empty Number Line:
T10 (Targeting the 10, partitioning and bridging through 10)

The steps in addition often bridge through a multiple of 10

e.g.

Children should be able to partition the 7 to relate adding the 2 first to target the 10 and then the 5.

$8 + 7 = 15$

O10 not recommended for written methods but can be used as a strategy in mental methods.

+ / = signs and missing numbers
Continue using a range of equations as in Stage 1 but with appropriate, larger numbers.
Extend to
$14 + 5 = 10 + \square$
and
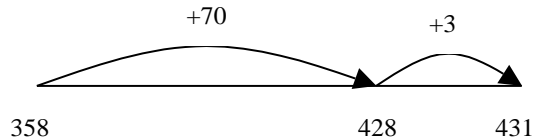$32 + \square + \square = 100$ $35 = 1 + \square + 5$

ADDITION GUIDELINES

Stage Four	Stage Five	Stage Six

Partition into hundreds, tens and ones and recombine
Either partition both numbers and recombine or partition the second number only e.g.
358 + 73 = 358 + 70 + 3
= 428 + 3
= 431
	Adding the least significant digits first
	<pre> 247
	+ 176
	    13 (7+6)
	   110 (40 + 70)
	   300 (200 + 100)
	   423</pre>
		Extend to numbers with at least four digits
		
		3587 + 675 = 4262
		
		<pre> 3587
		+ 675
		  4262
		  1 1 1</pre>

+70   +3

358   428   431

Stage Five (continued):

Working from <u>left to right</u>:
'Read' the answer from left to right, using knowledge of place value and referring to the value of each digit i.e.:"four hundred and twenty three"
<u>NOT</u> adding up columns for the final answer

Moving on to
<pre>   247
  + 376   (without use of brackets)
     13
    110
    500
    623</pre>

Moving on to a compact method
<pre>   247
  + 376
     623
     1 1</pre>
Working from <u>right to left</u>:
"7 + 6 is 13. Partition the 13 into 10 and 3, 'carry' the ten into the tens column, writing it as 1 to represent one ten."
n.b. the '1' can be written at the top or bottom of the calculation.
It is <u>NOT</u> "carry the 1"
Consolidation and practice of the previous key facts.

Stage Four (continued):

Horizontal Expansion

367 + 185 = 552

<pre> 367
+ 185
  400 (300+100)
  140 (60+80)
   12 (7+5)
  552</pre>

Moving on to

367 + 185 = 552

<pre> 367   (without use of brackets)
+ 185
  400
  140
   12
  552</pre>

Stage Six (continued):

Revert to expanded methods if the children experience any difficulty.

Partition into hundreds, tens, ones and decimal fractions and recombine
Either partition both numbers and recombine or partition the second number only e.g.
35.8 + 7.3 = 35.8 + 7 + 0.3
= 42.8 + 0.3
= 43.1

+7   +0.3

35.8   42.8   43.1
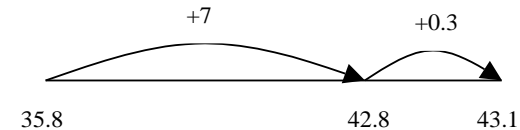
Extend to up to two places of decimals (same number of decimals places) and adding several numbers (with different numbers of digits).
<pre>   72.8
  + 54.6
   127.4
   1 1</pre>

ADDITION GUIDELINES		
Stage Four	**Stage Five**	**Stage Six**
Why most significant digit first and then least significant digit first? When adding and subtracting on a number line we start with the most significant digit first (e.g. add the tens then add the units). This is why horizontal expansion starts with the most significant digit first. Once the children are secure in this, it changes to adding the least significant digit first. This bridges the gap between these two stages (many children will only need to see it a few times to understand the relationship but others may need more experience at each stage) ***It is crucial to know or be able to derive key number facts TU + TU mentally or with jottings before progressing to Stage Five.*** **+ / = signs and missing numbers** Continue using a range of equations as in Stage 1 and 2 but with appropriate numbers. ***N.B. Please refer to the end of year expectation for the size and range of numbers to be using e.g. ThHTU, decimals, etc.***	***N.B. Please refer to the end of year expectation for the size and range of numbers to be using e.g. ThHTU, decimals, etc.***	**Extend to numbers with more than 4 digits or decimals with up to 3 places** $13.86 + 9.481 = 23.341$ $\begin{array}{r} 13.86 \\ + \ 9.481 \\ \hline 23.341 \\ \hline \scriptstyle 1 \ 1 \ 1 \end{array}$ $12350 + 4921$ $\begin{array}{r} 12350 \\ + \ 4921 \\ \hline 17271 \\ \hline \scriptstyle 1 \end{array}$ Revert to expanded methods if the children experience any difficulty. ***N.B. Please refer to the end of year expectation for the size and range of numbers to be using e.g. ThHTU, decimals, etc.***

End of Year Objectives for Addition

Yr1 – recall and jottings for U+U, T+U, T+T, TU+U (within 20 including 0)
Yr2 – TU+U, T+TU, TU+TU, U+U+U
Yr3 – mental methods for HTU + U, HTU+T, HTU+H; written methods for HTU+TU, HTU+HTU
Yr4 – written methods as above and ThHTU+ThHTU, U.t+U.t, £U.th+£U.th
Yr5 – written method for addition of numbers with more than four digits; 2 or more integers, decimals with 2dp e.g. 29.78 + 54.34
Yr6 – As above

Differentiation Steps for each Stage:
- Not crossing tens
- Crossing Tens
- Crossing Hundreds Only
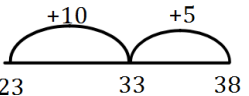- Crossing Tens and Hundreds
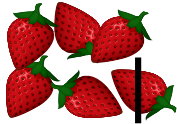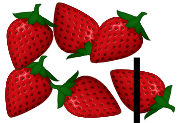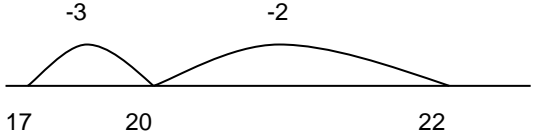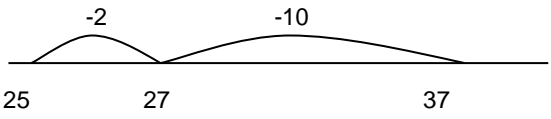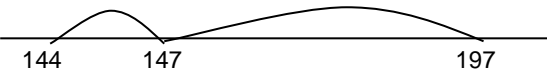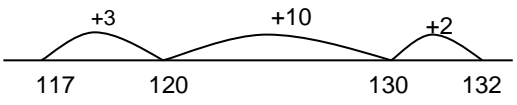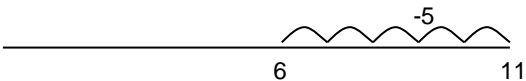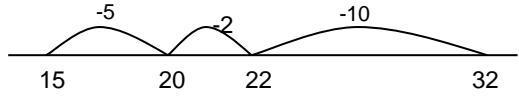
In addition:
- The number line must be modelled as an image to support calculation from Reception to Year 6.
- Jottings must be modelled as a clear image/strategy for mental calculation.
- If the calculation can be carried out mentally then do not give it to practise vertical calculation, e.g. TU + TU should not be calculated vertically.

Always present calculations horizontally in order to consider mental calculations first.
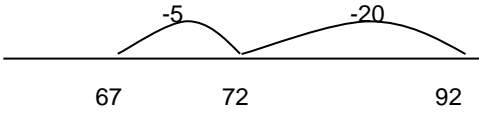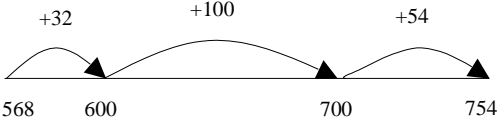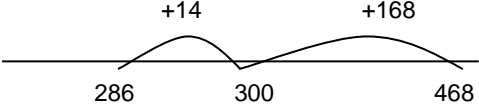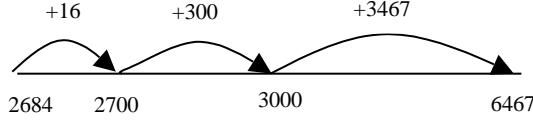
SUBTRACTION GUIDELINES

Stage One	Stage Two	Stage Three

Prerequisite skills (based on the practical)
Number bonds to 10
 	
Counting back from 20
 	
Find one less than a given number
 	
Subtract using quantities and objects 2 single digit numbers
 	
Count back to subtract single digit numbers
 	
There are two concepts linked to subtraction:
Subtract - where it is natural to count back to 'take away'
Find the difference – where the understanding of the vocabulary leads to using addition to count on [complementary addition].
 	
Understand subtraction as 'take away'
	There are two concepts linked to subtraction:
	Subtract - where it is natural to count back to 'take away'
	Find the difference – where the understanding of the vocabulary leads to using addition to count on [complementary addition].
	
	Use known number facts and place value to subtract
	
	Using knowledge of number bonds to subtract mentally from multiples of 10s e.g. $30 - 4$
	
	Using knowledge of number bonds to subtract mentally multiples of 10 from multiples of 10 e.g. if $7 - 4 = 3$ then $70 - 40 = 30$
	
	Using knowledge of number bonds to subtract mentally e.g. if $8 - 3 = 5$ then $28 - 3 = 25$
	
	Use of T10 for TU-U
	$22 - 5 = 22 - 2$
	    $= 20 - 3$
	
	   -3 -2
	
	17 20 22
	
	Use of J10 using multiples of 10
	Example: $80 - 30$
	
	Use of J10
	$37 - 12 = 37 - 10 - 2$
	        $= 27 - 2$
	        $= 25$
	
	  -2 -10
	
	25 27 37
		Use known number facts and place value to subtract
		Continue as in Stage 2 but with appropriate numbers
		e.g.$197 - 53 = 144$
		
		  -3 -50
		
		144 147 197
		
		Use counting on to find the difference
		$132 - 117 = 15$
		
		 +3 +10 +2
		
		117 120 130 132
		
		Secure knowledge in use of J10 and T10 to count back and find the difference.
		
		TU-TU, HTU-TU, HTU-HTU
		
		By the end of this stage children should know complements to 100. They can then use this knowledge to calculate HTU-TU, HTU-HTU.
		
		Subtract *mentally* a 'near multiple of 10' to or from a two-digit number
		Continue as in Stage 2 but with appropriate numbers e.g. $78 - 49$ is the same as $78 - 50 + 1$
		
		- = signs and missing numbers(inverse)
		
		Continue using a range of equations as in Stage 1 and 2 but with appropriate numbers.

SUBTRACTION GUIDELINES

Stage One	Stage Two	Stage Three
Use practical and informal written methods to support the subtraction of a one-digit number from a one digit or two-digit number and a multiple of 10 from a two-digit number. I have 11 toy cars. I lost 5 of them. How many are left? Start with bead strings / bars and move onto number lines below. -5 6 11 Use the vocabulary related to subtraction and symbols to describe and record subtraction number sentences (for the example above it would be 11 – 5 = 6) Recording by - drawing jumps on prepared lines / tracks Use practical resources to find the difference between two small numbers e.g. 6 and 7 Count on from smallest to largest number to find the difference where numbers are close in value. (e.g.9-7) **- = signs and missing numbers(inverse)** 7 - 3 = ☐ ☐ = 7 - 3 7 - ☐ = 4 4 = ☐ - 3 ☐ - 3 = 4 4 = 7 - ☐ ☐ - ∇ = 4 4 = ☐ - ∇	Use of T10 where necessary 32 - 17 -5 -2 -10 15 20 22 32 Subtraction for finding the difference using counting on e.g. 38 – 23 +10 +5 23 33 38 **- = signs and missing numbers(inverse)** Continue using a range of equations as in Stage 1 but with appropriate numbers. Extend to 14 + 5 = 20 - ☐ (inverse)	

SUBTRACTION GUIDELINES

(- = signs and missing numbers: Continue using a range of equations as in Stage 1 and 2 but with appropriate numbers.)

Stage Four	Stage Five	Stage Six

Find a small difference by counting up (relating to inverse)
 e.g. 5003 – 4996 = 7
 This can be modelled on an empty number line (see complementary addition). Children should be encouraged to use known number facts to reduce the number of steps.
 	
 Use known number facts and place value to subtract
 92 – 25 = 67
 	
 -5 $\quad$ -20
 	
 67 $\quad$ 72 $\quad$ 92
 	
 Counting on
 	
 Use of number facts to count up to find the difference (T10, T100). 754 – 568 = 186
 	
 $+32$ $\quad$ $+100$ $\quad$ $+54$
 	
 568 $\quad$ 600 $\quad$ 700 $\quad$ 754
 	
 For those children with a secure mental image of the number line they could record the jumps only:
 	
 754 – 568 = 186
 	
   754
 -568
    32 (600)
  100 (700)
    54 (754)
  186
	Counting on
	
	 Use of number facts to count up to find the difference (T10, T100). This is used in the context of inverse.
	
	 14 + 168 = 182 so:
	
	 468 – 286 = 182
	
	 $+14$ $\qquad$ $+168$
	
	 286 $\quad$ 300 $\quad$ 468
	
	 OR
	 754 - 286 = 468
	
	   754
	  -286
	    14 (300)
	  454 (754)
	  468
	
	 Reduce the number of steps to make the calculation more efficient.
	 Extend to 2 places of decimals
	
	 SUBTRACTION BY EXPANDED DECOMPOSITION (With higher attainers secure in number facts and use of the number line).
	
	 Subtracting with no repartitioning needed:
	
	   345 – 123
	
	   300 + 40 + 5
	
	 - (100 + 20 + 3)
	
	   200 + 20 + 2
	
	 Partitioning each number and working from right to left,
		Progress to 4 digit numbers
		
		 Teach on a number line first to subtract using T10, T100, T1000 (children should choose the most efficient method) either counting on or counting back.
		
		 e.g. 8000 – 2785 = 5215
		
		 To make this method more efficient, the number of jumps should be reduced to a minimum through children knowing:
		 ▪ Complements to 1, involving decimals to two decimal places (0.16 + 0.84)
		 ▪ Complements to 10, 100 and 100
		
		 Counting on
		
		 6467 – 2684 = 3783
		
		 $+16$ $\quad$ $+300$ $\quad$ $+3467$
		
		 2684 $\quad$ 2700 $\quad$ 3000 $\quad$ 6467
		 OR
		 6467 – 2684 = 3783
		
		   16 (2700) can be refined to 316 (3000)
		  300 (3000) 3467 (6467)
		 3467 (6467) 3783
		 3783
		 Reduce the number of steps to make the calculation more efficient.
		 Extend to 2 places of decimals
		
		 Subtraction by Standard Decomposition
		
		   346 – 128
		   3 $^3{\not4}^1$6
		  - 1 2 8
		    2 1 8

SUBTRACTION GUIDELINES
(- = signs and missing numbers: Continue using a range of equations as in Stage 1 and 2 but with appropriate numbers.)

Stage Four	Stage Five	Stage Six
Use known number facts and place value to subtract 6.1 – 2.4 = 3.7 -0.4 -2 3.7 4.1 6.1 **Use known number facts and place value to subtract** 0.5 – 0.31 = 0.19 -0.01 -0.3 0.19 0.2 0.5 ***N.B. Please refer to the end of year expectation for the size and range of numbers to be using e.g. ThHTU, decimals, etc.***	subtracting the bottom number form the top. Express each part as its value represented, i.e. "40 – 20". Moving onto subtracting with repartition of tens only: $252 - 114$ $200 + 50 + 2$ $-(100 + 10 + 4)$ _____ ? $200 + \overset{40}{\cancel{50}} + \overset{12}{\cancel{2}}$ $-(100 + 10 + 4)$ _____ $100 + 30 + 8$ Again, partitioning each number and working from right to left, subtracting the bottom number from the top. Where the subtraction is not possible i.e. 2 – 4 can't be done, the next value is "REPARTITIONED". So, "repartition 50 + 2 into 40 + 12". It is important to cross out the whole number and replace completely. Do NOT put a 'one in the air'! (It is not a 1, it is a 10.) Then repeat the subtraction process, this time "12 – 4 = 8" and "40 – 10 =30" ***N.B. Please refer to the end of year expectation for the size and range of numbers to be using e.g. ThHTU, decimals, etc.***	*It is still vital that the correct language of place value is used. The tens are REPARTITIONED (not "'borrow' a 1" and it is not "3 takeaway 1" but "300 takeaway/subtract/ minus 100").* *N.B. Please refer to the end of year expectation for the size and range of numbers to be using e.g. ThHTU, decimals, etc.*

End of Year Objectives for Subtraction

Year 1 – mentally subtract U-U, TU-U, TU- TU (up to 20 e.g. 15 – 12)
Year 2 - mentally TU-U, TU-multiple of 10, mentally with informal jottings TU-TU
Year 3 – subtract mentally, HTU – U, HTU – T, HTU – H, TU-U, TU-TU. Formal written methods for TU-TU, HTU-TU, HTU-HTU
Year 4 – as above and efficient written methods for ThHTU-ThHTU, ThHTU-HTU, U.t – U.t, £U.th-£U.th
Year 5 – Efficient written methods for subtraction of 2 integers with more than 4 digits e.g. 45230 - 12432 and decimals with up to 2dp e.g. 54.34-29.78
Year 6 – as above

Please note:
There are two concepts linked to subtraction:
Subtract - where it is natural to count back to 'take away'
Find the difference – where the understanding of the vocabulary leads to using addition to count on [complementary addition].

- Children should not move on to a written method if they are not completely confident with using a number line.
- Children will need to have had experience of different types of jumping on a number line e.g. T10 (target the ten), J10 (jump in 10s) and know how to partition numbers in different ways.
- These methods can also be easily applied, at different levels, to finding differences in values of money, measures and time.

**Always present calculations horizontally in order to consider mental calculations first.**

MULTIPLICATION

MENTAL STRATEGIES
Strategies to calculate the facts not yet recalled ARE essential:

× 2	double	÷ 2	halve
× 4	double-double	÷ 4	half and half again
× 8	double-double-double	÷ 8	half, half and half again

Model jottings for halving and doubling and use known facts and place value

½ of   6 4

½   ½

30   +   2
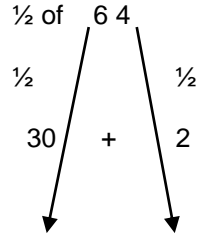
> *"Half of 6 tens or half of 60 is 3 tens or 30"*
> *"Half of 4 is 2."*

Where the number of tens (or hundreds) is odd and the fact unknown, use known facts to derive the new fact:

e.g.   ½ of   7 6

½

(60 + 10)

½   ½

30   +   5   +   3
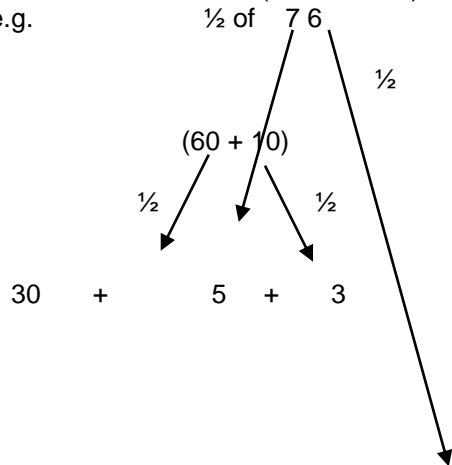
> **× 5**   *½ of × 10*
> **× 50**   *½ of × 100*
> **× 25**   *¼ of × 100 (or ½ and ½ again of × 100*
> **× 12**   *× 10 plus × 2 (double)*
> **× 15**   *× 10 plus ½ of × 10*

MULTIPLICATION GUIDELINES

Stage One	Stage Two	Stage Three

Stage One

Prerequisite skills (based on the practical)
Multiplication is related to known facts including doubling and counting groups of the same size.

3 + 3

E.g. use of dominoes and dice.

Counting using a variety of practical resources

Numicon and bead strings

Counting in 2s e.g. counting socks, shoes, animal's legs…
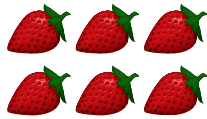
Counting in 5s e.g. counting fingers, fingers in gloves, toes…
Counting in 10s e.g. fingers, toes…

Stage Two

x = signs and missing numbers

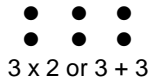|	
7 x 2 = ☐	☐ = 2 x 7
7 x ☐ = 14	14 = ☐ x 7
☐ x 2 = 14	14 = 2 x ☐
☐ x ∇ = 14	14 = ☐ x ∇

Arrays and repeated addition

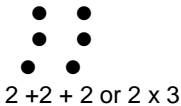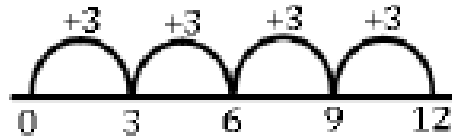Looking at rows
3 + 3
2 groups of 3

Looking at rows
2 + 2 + 2
3 groups of 2

3 x 2 or 3 + 3

2 +2 + 2 or 2 x 3

If the calculation is 3 x 4 for example, children should understand that this means 3 + 3 + 3 + 3. Children should also understand the commutative law and be able to use 4 x 3.

Stage Three

x = signs and missing numbers
Continue using a range of equations as in Stage 2 but with appropriate numbers.

Arrays and repeated addition
Continue to understand multiplication as repeated addition and continue to use arrays and number lines (as in Stage 2).

Use known facts and place value to carry out simple multiplications

Partition
23 x 3 =

X	20	3
3	3 x 20 = 60	3 x 3 = 9

Moving on to:

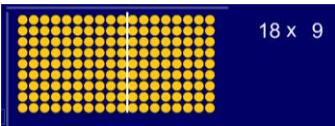| x	10	6
10	10 x 10= 100	10 x 6 = 60
7	7 x 10 = 70	7 x 6 = 42

100 + 60 + 70 + 42 = 272

At the end of Stage 3 the children should know their 12 x 12 times tables.

MULTIPLICATION GUIDELINES		
Stage One	**Stage Two**	**Stage Three**
Pictures / marks There are 2 socks in a pair How many socks are there in 3 pairs? *The above is required before moving on to Stage 2.*	**Partitioning** Children need to be secure with partitioning numbers into 10s and 1s and partitioning in different ways: 6 = 5 + 1 so e.g. Double 6 is the same as double five add double one. $\boxed{2 \mid 3}$ X 3 = 69 $\boxed{2 \mid 0}$ X 3 = 60 3 x 3 =9 *At the end of Stage 2 the children should use the above strategies, as well as doubles of multiples of 5 and knowing the 2, 3, 4, 5, 6, 8 and 10 times tables from memory.*	

MULTIPLICATION GUIDELINES

Stage Four	Stage Five	Stage Six

Stage Four

x = signs and missing numbers
Continue using a range of equations as in Stage 3 but with appropriate numbers

Partition
Continue to use arrays:

18 x 9

18 x 9 = 162

18 x 9 = (10 x 9) + (8 x 9) = 162

Use Multiplication array ITP to model partitioning into tens and ones, using the familiar visual pattern of 5s.

OR

Use the grid method of multiplication (as below)

36 x 27 =

x	30	6
20	20 x 30= 600	20 x 6 = 120
7	7 x 30 = 210	7 x 6 = 42

600 + 120 + 210 + 42 = 972

Stage Five

Partition
47 x 6 = 282

47 x 6 = (40 x 6) + (7 x 6) = 282

OR

Use the grid method of multiplication (as below)

Grid method
72 x 38 is approximately 70 x 40 = 2800

Remember, always present calculations horizontally in order to consider mental calculations first.

Again, if the calculation should be possible mentally then do not give it to practise vertical calculation, e.g. 23 x 15 should not be calculated vertically. Consider use of numbers carefully. Avoid numbers which involve x 2, x 4, x 5, x 8 which can be solved mentally using known facts.

382 x 23 =

x	300	80	2
20	20 x 300 = 6000	20 x 80 = 1600	20 x 2 = 40
3	3 x 300= 900	3 x 80 = 240	3 x 2 = 6

6000 + 1600 + 900 + 240 + 240 + 40 + 6 = 8986
6000 + 2500 + 480 + 46 = 8000+ 980 +46
It is important to write the calculation in the grid for both the pupil and teacher to be able to identify errors made in multiplication facts or in the calculating the process.
It is also a reminder that the area of the rectangle is being calculated and the system is clear.

Stage Six

Use the grid method of multiplication (as below)
Grid method
372 x 24 is approximately 400 x 20 = 8000

Extend to decimals with up to two decimal places.

The recording is reduced further, with carry digits recorded below the line.

```
   38
 x  7
  266
    5
```

Children who are already secure with multiplication for TU × U and TU × TU should have little difficulty in using the same method for HTU × TU or applying decimals.

Long multiplication

124 × 26 becomes

Answer: 3224

Stage Four	Stage Five	Stage Six
	Where possible, use mental calculation strategies to calculate the total e.g. looking for known facts or adding the largest number first. **Use Multiplication grid ITP** to assess understanding and application of the grid method by 'hiding' the question parts and 'revealing' some of the answer parts. You can extend to using the grid method to multiply decimals. **Expanded Column Multiplication** *Children should describe what they do by referring to the actual values of the digits in the columns. For example, the first step in 382 × 23 is 'three hundreds multiplied by twenty', not 'three times two', although the relationship 3 × 2 should be stressed.*	

MULTIPLICATION GUIDELINES		
Stage Four	**Stage Five**	**Stage Six**
| | **Most significant first**<br><br>382 x 23 =<br><br>```
  300 + 80 + 2       300 + 80 + 2
X       20 + 3     X       20 + 3
     6000  (20 x 300)     6000
     1600  (20 x 80)      1600
       40  (20 x 2)         40
      900  (3 x 300)       900
      240  (3 x 80)        240
        6  (3 x 2)           6
     8786                  8786
```<br><br>**Least significant first**<br><br>382 x 23 =<br><br>```
  300 + 80 + 2       300 + 80 + 2
X       20 + 3     X       20 + 3
        6  (3 x 2)           6
      240  (3 x 80)        240
      900  (3 x 300)       900
       40  (20 x 2)         40
     1600  (20 x 80)      1600
     6000  (20 x 300)     6000
     8786                  8786
``` | |

End of Year Objectives for Multiplication

Year 1 – practical problems that combine groups of 2, 5 or 10
Year 2 - represent multiplication as repeated + and arrays. Practical and informal written methods and vocabulary used to support multiplication alongside known facts and mental strategies. Understand and use '3 for free' for x and ÷ of the 2, 3,4,5,6, 8 and 10 times-tables.
Year 3 – Describe the effect of Ux10, TUx10, Ux100, TU x 100. Practical and informal written methods for TU x U.
Year 4 – Derive and recall x and ÷ facts up to 12 x 12 and '3 for free' facts. Multiply numbers to 1000 by 10 and 100. Formal written layout and explain TU/HTU x U.
Year 5 – mentally multiply TU x U. Multiply whole numbers and decimals by 10, 100 and 1000. Formal written methods to multiply ThHTU x U, ThHTU x TU, U.t x U
Year 6 – mentally calculate TU x U, U.t X U. Formal written methods to multiply up to 4 digit by 2 digit and one digit with up to 2 decimal places.

As with addition and subtraction, before progressing through the stages of calculation:
Learning
- It is crucial to know or be able to derive key number facts:
⇨ Understand and use doubling and halving
⇨ x/÷ 10 (as moving a place to the left/right <u>NOT</u> "add a zero" etc.!!)

- Place value and partitioning MUST be clearly understood and explained using the appropriate mathematical vocabulary.

Teaching
- The number line and the use of arrays must be modelled as images to support calculation from Reception to Year 6.
- Jottings must be modelled as a clear image/strategy for mental calculation.
- If the calculation should be possible mentally then do not give it to practise vertical calculation, e.g. 23 x 15 should not be calculated vertically. Consider use of numbers carefully.

Always present calculations horizontally in order to consider mental calculations first.

DIVISION GUIDELINES

Stage One	Stage Two	Stage Three
Prerequisite skills (based on the practical) Understanding the language of half in different contexts. Know halves of even numbers up to 10. **Sharing** Requires secure counting skills -see counting and understanding number strand Develops importance of one-to-one correspondence See appendix for additional information on x and ÷ and aspects of number Sharing – 6 sweets are shared between 2 people. How many do they have each? Practical activities involving sharing, distributing cards when playing a game, putting objects onto plates, into cups, hoops etc. **Grouping** Sorting objects into 2s / 5s/ 10s etc. How many pairs of socks are there? There are 10 bulbs. Plant 5 in each pot. How many pots are there? Jo has 10 Lego wheels. How many bicycles can she make?	**÷ = signs and missing numbers** $6 \div 2 = \square$ $\square = 6 \div 2$ $6 \div \square = 3$ $3 = 6 \div \square$ $\square \div 2 = 3$ $3 = \square \div 2$ $\square \div \nabla = 3$ $3 = \square \div \nabla$ **Grouping** Link to counting and understanding number strand Count up to 100 objects by grouping them and counting in tens, fives or twos;… Find one half, one quarter and three quarters of shapes and sets of objects $15 \div 5$ can be modelled as: There are 15 strawberries. How many people can have 5 each? How many 5s make 15? $15 \div 5$ can be modelled as repeated subtraction In the context of money count forwards and backwards using 2p, 5p and 10p coins Practical grouping e.g. in PE 12 children get into teams of 4 to play a game. How many teams are there? Children should know that division is not commutative.	**÷ = signs and missing numbers** Continue using a range of equations as in Stage 2 but with appropriate numbers. **Understand division as sharing and grouping** $24 \div 3$ can be modelled as: Sharing – 24 shared between 3 OR Grouping - How many 3's make 24? **Remainders** $23 \div 4 = 5r3$ Sharing - 23 shared between 4, how many left over? Grouping – How many 4's make 23, how many left over? e.g.

DIVISION GUIDELINES

Stage Four	Stage Five	Stage Six

÷ = signs and missing numbers
Continue using a range of equations as in Stage 2 but with appropriate numbers.
Sharing and grouping
60 ÷ 12 can be modelled as:
grouping – 12 subtracted repeatedly from 60 on a no. line, leading to subtracting 'groups' of 12.
sharing – sharing among 12, the number given to each person.
Remainders
 	
41 ÷ 4 = 10 r1
 	
+40
+1
10 groups
 	
41 = (10 x 4) + 1
 	
Pencil and paper procedures- Chunking.
72 ÷ 5 lies between 5 0 ÷ 5 = 10 and 100 ÷ 5 = 20
 	
Key Facts
1 x 5 = 5
2 x 5 = 10
5 x 5 = 25
10 x 5 = 50
 	
    72
- 50   (10 groups)
   22
- 20   (4 groups)
    2
 	
Answer : 14 remainder 2
	Sharing and grouping
	
	Continue to understand division as both sharing and grouping (repeated subtraction).
	Remainders
	
	Key Facts
	1 x 7 = 7
	2 x 7 = 14
	5 x 7 = 35
	10 x 7 = 70
	
	Pencil and paper procedures-Chunking
	
	256 ÷ 7 lies between 210 ÷ 7 = 30 and 280 ÷ 7 = 40
	
	    256
	- 210 7 x 30
	    46
	- 42 7 x 6
	    4
	
	Answer: 36 remainder 4
	Quotients expressed as fractions or decimal fractions
	61 ÷ 4 = 15 ¼ or 15.25
	Also, Short Division for More Able Children
	
	432 ÷ 5 becomes
	
	   8 6 r 2
	5
		4 3 2
		    ³
		
		Considering each column starting from the left.
		
		Answer: 86 remainder 2
			Sharing, grouping and remainders as Stage Five
			Pencil and paper procedures-Chunking
			
			977 ÷ 36 is approximately 1000 ÷ 40 =
			    977
			- 720 36 x 20
			   257
			- 180 36 x 5
			    77
			- 72 36 x 2
			    5
			
			Answer: 27 $^5/_{36}$
			
			Key Facts
			1 x 36 = 36
			2 x 36 = 72
			5 x 36 = 180
			10 x 36 = 360
			
			Pencil and Paper procedures- Short Division Method
			
			quotient
			divisor 5
				847	dividend
					
					496 ÷ 11 becomes
					
					   4 5 r 1
					1 1
						4 9 6
						     ⁵
						
						Answer: 45 $\frac{1}{11}$
						
						Both methods above are necessary at this stage, to deal with the wide range of problems experienced at Stage Six.

End of Year Objectives for Division

Year 1 – practical problems that share into equal groups of 2, 5 or 10.

Year 2 – derive and recall division facts for 2, 5 or 10, represent division as repeated subtraction (grouping) and sharing. Practical and informal written methods and vocabulary used to support division, including remainders. To know that division is not commutative.

Year 3 – Practical and informal written methods for TU÷U. Understand and use '3 for free' for x and ÷ of the 2, 3, 4, 5, 6, 8 and 10 times-tables. Round remainders up or down, depending on the context.

Year 4 – Derive and recall x facts up to 12x12 and apply '3 for free' facts. Divide numbers to 1000 by 10 and 100. Develop and use formal written layouts to record.

Year 5 – Divide whole numbers and decimals by 10, 100 and 1000. Divide numbers up to 4 digits by a one digit number using the formal written methods for division and interpret remainders appropriately for the context.

Year 6 – Divide numbers up to 4 digits by a 2 digit whole number using the formal written method of long division interpreting remainders as fractions, decimals, etc. Divide numbers up to 4 digits by a two digit number using the formal written methods for division and interpret remainders appropriately for the context.

As with multiplication, before progressing through the stages of calculation:
Learning
- It is crucial to know or be able to derive key number facts:
  - ⇨ Understand and use doubling and halving
  - ⇨ x/÷ 10 (as moving a place to the left/right <u>NOT</u> "add a zero" etc.!!)

- Place value and partitioning MUST be clearly understood and explained using the appropriate mathematical vocabulary.

Teaching
- The number line and the use of arrays must be modelled as images to support calculation from Reception to Year 6.
- Jottings must be modelled as a clear image/strategy for mental calculation.
- If the calculation should be possible mentally then do not give it to practise vertical calculation, e.g. 24 ÷ 3 should not be calculated using short division. Consider use of numbers carefully.