



Calculations Policy

Approved by Headteacher on: 20th February 2019

Review Date: Spring 2021

Contents of the policy

1.	Key Messages.....	p2
2.	Mathematical language.....	p5
3.	Early Years Foundation Stage.....	p6
4.	Addition Years 1-6.....	p10
5.	Subtraction Years 1-6.....	p18
6.	Multiplication Years 1-6.....	p26
7.	Division Years 1-6.....	p35

Mathematics Mastery

At the center of this approach is the belief that all children have the potential to succeed. They should have access to the same curriculum and be enabled to deepen their conceptual understanding by tackling challenging and varied problems. Similarly, with calculation strategies, must not simply learn rote procedures but demonstrate their understanding through the concrete and pictorial. This policy outlines the different calculation strategies that should be being taught between year 1 and year 6.

Background

The 2014 national curriculum differed from its predecessor in many ways. One of the key differences is the level of detail included, indicating what children should be learning and when. This is suggested content for each year group but schools have been given autonomy to introduce content earlier or later, with the expectation that by the end of each key stage the required content has been covered. In many ways these specific objectives make it easier for teachers to plan a coherent approach to the development of pupils' calculation skills.


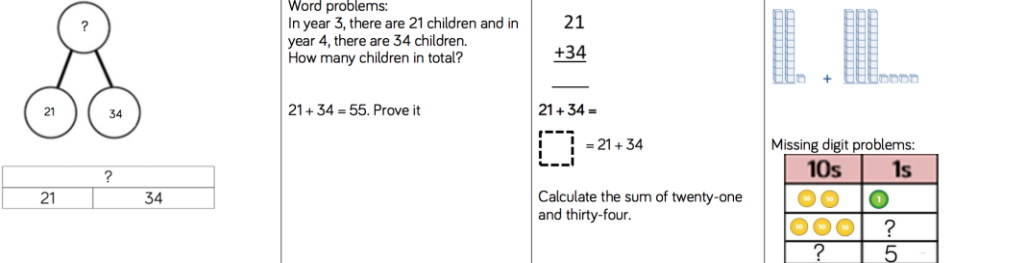

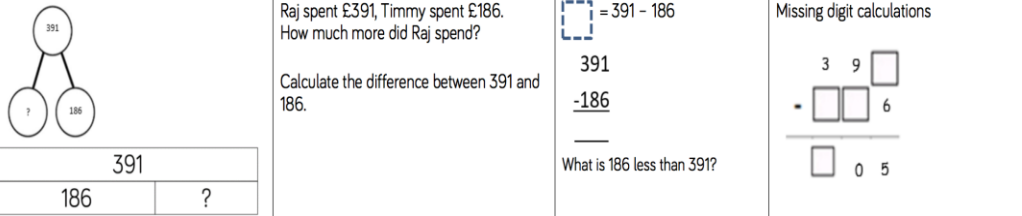
Mathematical Language

The 2014 national curriculum is explicit in articulating the importance of children using the correct mathematical language as a central part of their learning and reasoning. Indeed, in certain year groups, there is a requirement for children to extend their language around certain concepts. Therefore, it is essential that teaching the calculations presented in this policy is accompanied with the appropriate and precise mathematical vocabulary. When new vocabulary is introduced it should be done so in a suitable context and explained carefully. High expectations of the mathematical language used is essential, with teachers only accepting what is correct. This is explicitly stated in the 2014 Maths Programme of Study: the quality and variety of language that pupils hear and speak are key factors in developing their mathematical vocabulary and presenting a mathematical justification, argument or proof.

How to use the policy

The mathematics policy is a guide for ALL staff at Barley Lane Primary School and has been adapted from the White Rose Maths Hub. It is set out a progression of mathematical skills to encourage a flexible approach to teaching and learning. It is expected that teachers will use their professional judgement in regards to consolidating existing skills and moving onto the next concept. The focus must always remain on breadth and depth as opposed to accelerating through concepts. Children should not be extending with new learning before they are ready. This is intended to complement the White Rose Maths Hub planning blocks which we are using to plan our lessons and delivery of the curriculum. Teachers can use the resources that they wish to as this policy does not recommend one over the other but, in fact, encourages that a variety of resources are used.

1. Key Messages

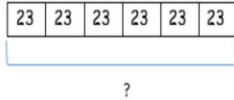
	Key language	When designing activities for addition we want varied presentation of question types. Here are just some of the ways that we vary asking some of the same question.	Concepts
<p>Addition</p> 	<p>sum, total, parts and whole, add, altogether, more, is equal to and is the same as</p>	<p>Conceptual variation; different ways to ask children to solve 21 + 34</p> 	<p>In addition, the number line must be modelled, and us as a representation all the way from foundation stage to year 6.</p> <p>Jottings must be modelled as clear image/strategy for mental calculations.</p> <p>If the calculation can be carried out mentally then do not give it to practice vertical calculation.</p> <p>Always present calculations horizontally at first so that mental calculations can be considered</p>
<p>Subtraction</p> 	<p>take away, less than, the difference, subtract, minus, fewer and decrease.</p>	<p>Conceptual variation; different ways to ask children to solve 391 - 186</p> 	<p>There are two concepts linked to subtraction:</p> <p>Subtract - where it is natural to count back to take away</p> <p>Find the difference - where the understanding of the vocabulary leads to the using of addition to count on</p> <p>Children should not move on to a written method until they are entirely confident with using the number line.</p> <p>Children need to have experience with different sort of jumps on the number line and know how to partition numbers in a variety of different ways.</p> <p>Always present calculations horizontally so that a mental calculation can be considered.</p>

Multiplication



double, times, multiplied by, the product of, groups of, lots of, equal groups.

Conceptual variation; different ways to ask children to solve 6×23



Mai had to swim 23 lengths, 6 times a week.
How many lengths did she swim in one week?

With the counters, prove that $6 \times 23 = 138$

Find the product of 6 and 23

$$6 \times 23 =$$

$$\square = 6 \times 23$$

$$\begin{array}{r} 6 \\ \times 23 \\ \hline \end{array} \quad \begin{array}{r} 23 \\ \times 6 \\ \hline \end{array}$$

What is the calculation?
What is the product?



It is crucial to be able to know and derive key number facts

It is vital that the children are confident with doubling and halving.

They need to know that when they are multiplying by ten they are moving the number as place to the left and NOT adding a zero

Both number lines and arrays should be used as models and representations all the way from reception till year 6.
Jottings must be modelled as a clear image/strategy for mental calculations.
If the question should be possible mentally then do not give it to practice vertical multiplication.
Always present calculations horizontally so that a mental calculation can be considered.

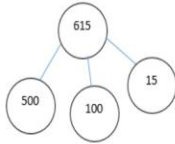
Division



share, group, divide, divided by and half.

Conceptual variation; different ways to ask children to solve $615 \div 5$

Using the part whole model below, how can you divide 615 by 5 without using short division?



I have £615 and share it equally between 5 bank accounts. How much will be in each account?

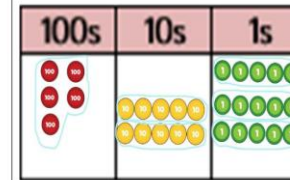
615 pupils need to be put into 5 groups. How many will be in each group?

$$5 \overline{)615}$$

$$615 \div 5 =$$

$$\square = 615 \div 5$$

What is the calculation?
What is the answer?



It is crucial for the children to know, or be able to derive key number facts.

The children should have a secure understanding of doubling and halving.

They need to know that when they are dividing by ten they move the number to the right as opposed to 'adding a zero.'

The number line and array must be used as models and representations from reception until year 6.
Jottings must be modelled as a clear image/strategy for mental calculation.
If the question should be possible mentally then do not give it to practice formal division.
Always present calculations horizontally so that a mental calculation can be considered.

2. Mathematical Vocabulary

Correct terminology	Incorrect terminology
Ones	Units
Is equal to (is the same as)	Equals
Zero	Oh (the letter o)
Exchange/Exchanging Regrouping Repartitioning	Stealing Borrowing
Calculation Equation	Generic term of 'sum' or 'number sentence.'
Bar method/Thinking blocks	Bar model is preferable over thinking blocks

3. Early Years Foundation Stage Addition

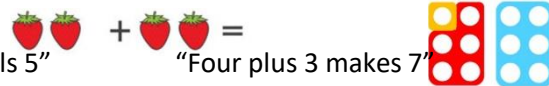
Maths for young children should be meaningful. Where possible, concepts should be taught in the context of real life. Have an understanding of what “more” means and be able to say what is one more than a given number.

1. Children begin to combine groups of objects or pictures and use concrete apparatus.
2. Solve simple problems using fingers and introduce Numicon when appropriate.



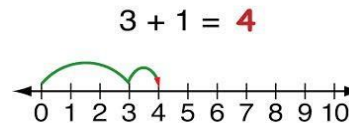
3. Children make a record in pictures, words, Numicon shapes or symbols of addition activities already carried out.

4. Children are encouraged to read number sentences



e.g. “Three add two equals 5” “Four plus 3 makes 7”

5. Construct number sentences verbally, or by using cards to go with practical activities.
6. Number lines can be used alongside practical apparatus to solve addition calculations and word problems. Children “jump” along the number line to “count on”.



Key Vocabulary: Games and songs can be a useful way to begin using the vocabulary involved in addition.

add, more, plus, makes, total, altogether, score, double, one more, two more, ten more. how many more to make...? how many more is ... than ...?

Key skills for addition in Foundation Stage:

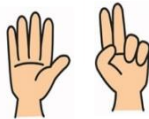
- Select the correct numeral to represent 1 to 5, then 1 to 10 objects.
- Count an irregular arrangement of up to ten objects.
- Estimate how many objects they can see and check by counting them.
- Use the language of ‘more’ and ‘fewer’ to compare two sets of objects.
- Find the total number of items in two groups by counting all of them.
- Say the number that is one more than a given number.
- Find one more from a group of up to five objects, then ten objects.
- In practical activities and discussion, begin to use the vocabulary involved in addition
- Record, using marks that they can interpret and explain.
- Begin to identify own mathematical problems based on own interests and fascinations

Subtraction – Foundation Stage

1. Have an understanding of what “less” means and be able to say what is one less than a given number.
2. Children begin to use objects, pictures and concrete apparatus to relate subtraction to taking away and counting how many objects are left.
3. Solve simple problems using fingers and introduce Numicon where appropriate.



$$5 - 3 = 2$$

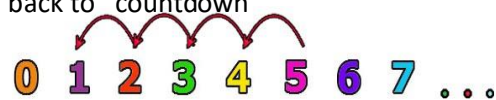


4. Children make a record in pictures, words, Numicon shapes or symbols of subtraction activities already carried out.



10 take away 5 leaves 5

5. Children are encouraged to read number sentences aloud in different ways
e.g. “Five subtract one leaves four” “Six take away 3 equals 3”
6. Construct number sentences verbally or using cards to go with practical activities.
7. Number lines can be used alongside practical apparatus to solve subtraction
8. calculations and word problems “jump” back to “countdown” the number line.



$$5 - 4 = 1$$

Key Vocabulary: Games and songs can be a useful way to begin using the vocabulary involved in subtraction: e.g. Five Little Men in a Flying Saucer, Ten Green Bottles, and Five Currant Buns
take, take away, leave, subtract, minus, equals, number sentence, count back, one less, two less, ten less how many are left / left over? How many have gone? How many fewer is ... than ...?

Key skills for subtraction in **Foundation Stage**:

- Select the correct numeral to represent 1 to 5, then 1 to 10 objects.
- Count an irregular arrangement of up to ten objects.
- Estimate how many objects they can see and check by counting them.
- Use the language of ‘more’ and ‘fewer’ to compare two sets of objects.
- Say the number that is one less than a given number.
- Find one less from a group of up to five objects, then ten objects.
- In practical activities and discussion, begin to use the vocabulary involved in subtraction
- Record, using marks that they can interpret and explain.
- Begin to identify own mathematical problems based on own interests and fascinations

Multiplication – Foundation Stage

The link between addition and multiplication can be introduced through doubling and reinforced through repeated addition of the same number.

1. Children begin with mostly pictorial representations.

 How many groups of 2 are there? 3 groups of 2 = 6

2. Real life contexts and use of practical **equipment to count in repeated groups of the same size.**



How many wheels are there altogether?



How much money do I have?

3. Count in twos, fives and tens, both aloud and with objects, such as Numicon or other concrete apparatus.

2, 4, 6, 8, 10, 12

4. Children are encouraged to read number sentences aloud in different ways

e.g. “Five groups of two makes ten” “Three lots of two makes six”



5. Children are given multiplication problems set in a real life context and are encouraged to visualise the problem.

e.g. “How many fingers on two hands?” “How many sides on three triangles?”



“How many legs on four ducks?”

Key Vocabulary: lots of, groups of, times, repeated addition, double, combine, twos, fives, tens

Key skills for multiplication in **Foundation Stage**:

- Select the correct numeral to represent 1 to 5, then 1 to 10 objects.
- Count an irregular arrangement of up to ten objects.
- Estimate how many objects they can see and check by counting them.
- Find the total number of items in two groups by counting all of them.
- Record, using marks that they can interpret and explain.
- Begin to identify own mathematical problems based on own interests and fascinations.

Division – Foundation Stage

1. Division can be introduced through halving or sharing an equal amount into 2 groups.



2. Children begin with mostly pictorial representations linked to real life contexts:



Grouping Model
Mum has 6 socks. She grouped them into pairs. How many pairs did she make?



Sharing Model
I have 10 sweets. I want to share them with my friend. How many will we have each?

Children need to see and hear representations of division as both grouping and sharing.

3. Children have a go at recording the calculation that has been carried out:
e.g. by drawing pictures in groups or by arranging concrete apparatus into groups.



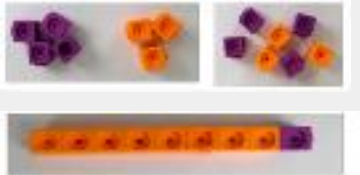
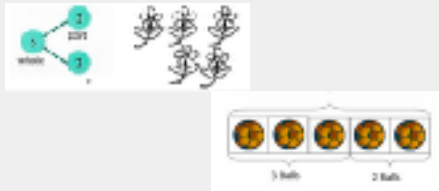


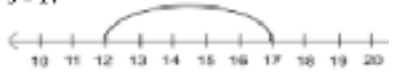
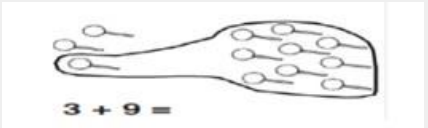
12 shared equally by 3 is 4

Key Vocabulary: halve, share, share equally, one each, two each, three each, group in pairs / threes / tens, equal groups of, in equal parts, left, left over

Key skills for division in **Foundation Stage**:

- Select the correct numeral to represent 1 to 5, then 1 to 10 objects.
- Count an irregular arrangement of up to ten objects.
- Estimate how many objects they can see and check by counting them.
- Record, using marks that they can interpret and explain.
- Begin to identify own mathematical problems based on own interests and fascinations.

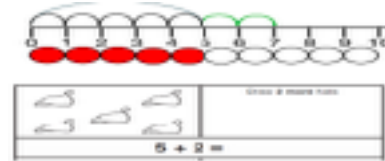
Addition - Year 1

Objective and strategy	Concrete	Pictorial	Abstract
<p>Combining two parts to make a whole: part whole model</p>	<p>Use part whole model. Use cubes to add two numbers together as a group or, alternatively, in a bar</p> 	<p>Use pictures and images to add two numbers together as a group or, alternatively, in a bar.</p> 	<p>Use the part-whole diagram (as shown below) to move the children into the abstract.</p> 
<p>Starting at the bigger number and counting on</p>	<p>Start with the larger number on the bead string and then count on the smaller number one by one to get the answer</p> 	<p>Start at the larger number on the number line and count on in ones or in one jump to find the answer</p> <p>$12 + 5 = 17$</p> 	<p>Place the larger number in your head and count on the smaller number to find your answer ($5 + 12 = 17$)</p>
<p>Regrouping to make 10 (this is essential for the children to know in preparation for column addition later)</p>	<p>Start with the bigger number and use the smaller number to make ten (if possible use a ten frame or numicon)</p>	<p>Use pictures or a number line. Regroup (or partition) the smaller number to make ten.</p> 	<p>$7 + 4 = 11$</p> <p>If I am at 7 how many more do I need to make ten? How many more do I add on now?</p>



Represent and use number bonds and related subtraction facts within 20.

2 more than 5.

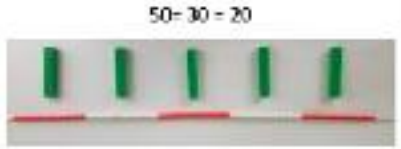


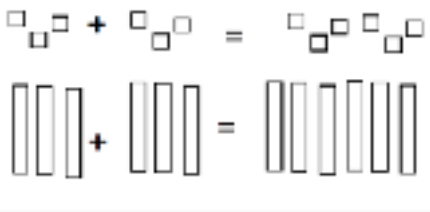
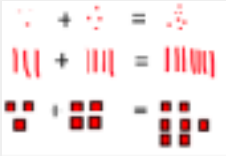
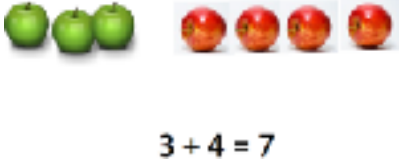

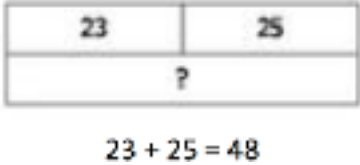


Emphasis should be on the mathematical language


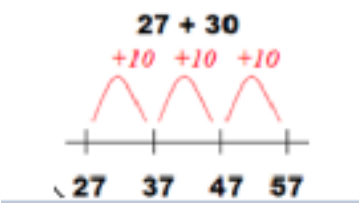

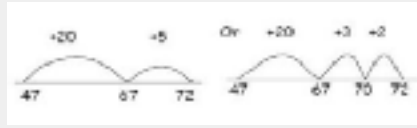
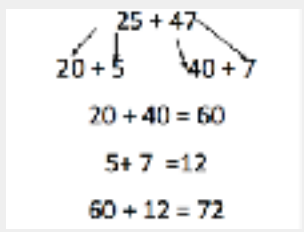
'One more than 5 is equal to 6.' *'2 more than 5 is 7.'*
'8 is 3 more than 5.'



Addition - Year 2

Objective and strategy	Concrete	Pictorial	Abstract
Adding multiples of ten	$50 = 30 + 20$ (Model using dienes and bead strings) 	Use representations for base ten 	$20 + 30 = 50$ $70 = 50 + 20$ $40 + \underline{\quad} = 60$
Use known number facts	Children explores ways of making numbers within 20		$\square + 1 = 16$ $16 - 1 = \square$ $1 + \square = 16$ $16 - \square = 1$
Use known facts		Children draw representations of hundreds, tens and ones. 	$3 + 4 = 7$ leads to $30 + 40 = 70$ leads to $300 + 400 = 700$
Introduction of the bar model			



Addition Year 2 cntd	Concrete	Pictorial	Abstract				
Add a two digit number and ones	Use ten frame to make 'magic ten.' Children explore the pattern - $17 + 5 = 22$	Use part whole and number line to model	Explore related facts with the children <div style="border: 1px solid black; padding: 5px; width: fit-content;"> <p>Explore related facts</p> <p>$17 + 5 = 22$</p> <p>$5 + 17 = 22$</p> <p>$22 - 17 = 5$</p> <p>$22 - 5 = 17$</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td colspan="2" style="text-align: center;">22</td></tr> <tr><td style="width: 50px;">17</td><td style="width: 50px;">5</td></tr> </table> </div>	22		17	5
22							
17	5						
Add a two digit number and tens	Explore with the children that the ones digit does not change  <p style="text-align: center;">$25 + 10 = 35$</p>	 <p style="text-align: center;">$27 + 30 = 57$</p>	$27 + 10 = 37$ $27 + 20 = 47$ $27 + \square = 57$				
Add 2 two digit numbers	Model using dienes, place value counters and numicon 	Use number line and bridge ten (using part whole if necessary) 	 <p style="text-align: center;">$25 + 47 = 72$</p>				

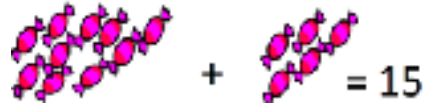


Add 3 one digit numbers

Combine to make ten first (if possible) or bridge ten and then add third digit.



Regroup and draw representation.

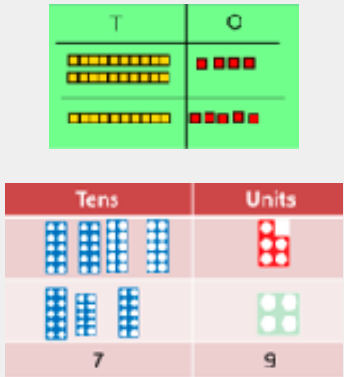

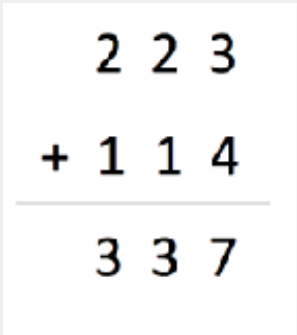
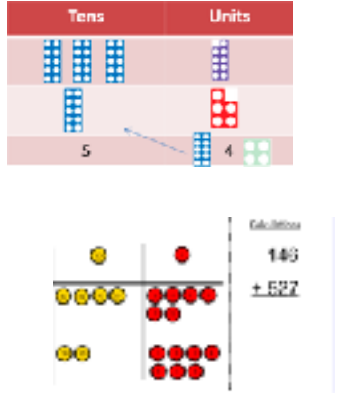
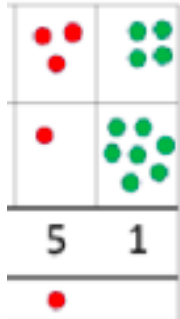
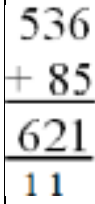


Combine the two numbers that make/ bridge ten - then add on the third

$$\begin{aligned} 4 + 7 + 6 &= 10 + 7 \\ &= 17 \end{aligned}$$

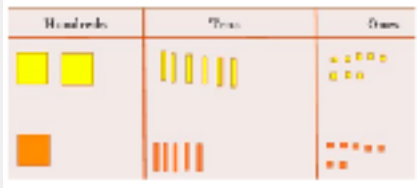
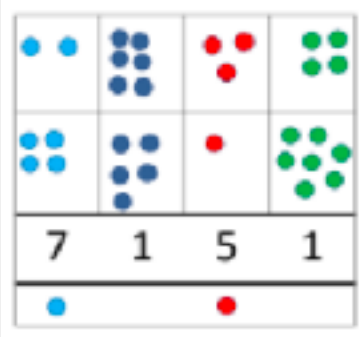
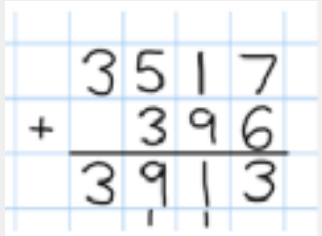
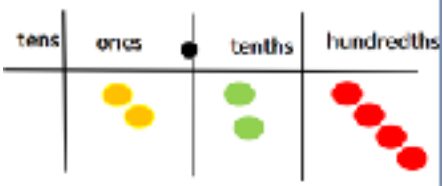
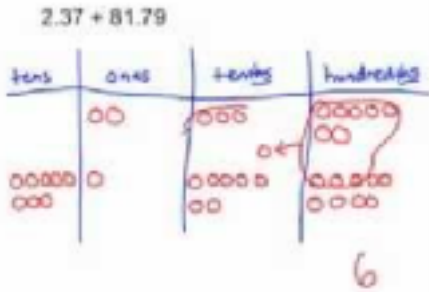
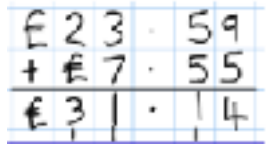


Addition - Year 3

Objective and strategy	Concrete	Pictorial	Abstract
<p>Column addition (no regrouping). Friendly numbers to work with</p> <p>Add two or three 2 and 3 digit numbers</p>	<p>Model using dienes or numicon. Add together the ones first and then the tens.</p> 	<p>Children move to drawing the counters using a tens and one frame</p> 	<p>Add the ones first, then the tens and then the hundreds.</p> 
<p>Column addition with regrouping</p>	<p>Exchange ten ones for a ten. Model using numicon and counters.</p> 	<p>Children can draw a representation of the grid to further support and develop their understanding carrying the ten underneath the line</p> 	<p>Start by partitioning the numbers before formal column to demonstrate the exchange</p> $\begin{array}{r} 20 + 5 \\ 40 + 8 \\ 60 + 13 = 73 \end{array}$ 



Addition - Years 4-6

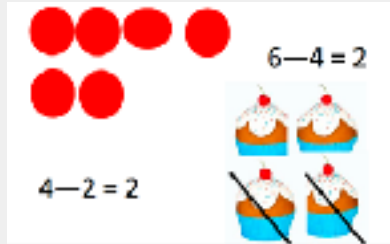
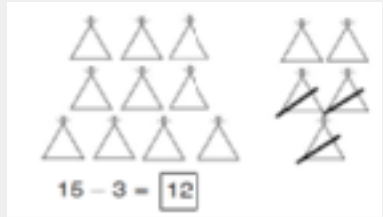
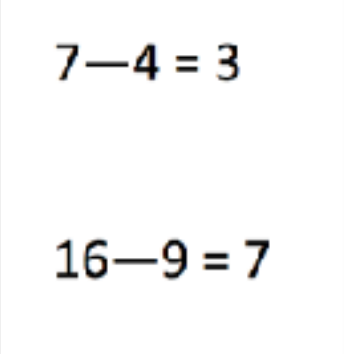

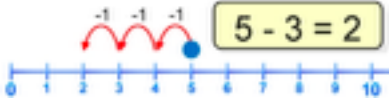
Objective and strategy	Concrete	Pictorial	Abstract
<p>Year 4 - add numbers with up to 4 digits</p>	<p>Children continue to use dienes or counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand</p> 	<p>Draw representations using a grid</p> 	<p>Continue from previous work to carry hundreds as well as tens. Relate to money and measures</p> 
<p>Year 5 - add numbers with more than 4 digits.</p> <p>Add decimals with two decimal places, including in a money context</p>	<p>As year 4.</p> <p>Introduce using counters for numbers after the decimal point and model exchange for addition</p> 	<p>$2.37 + 81.79$</p> 	<p>72.8 $+ 54.6$ <u>127.4</u> 11</p> 



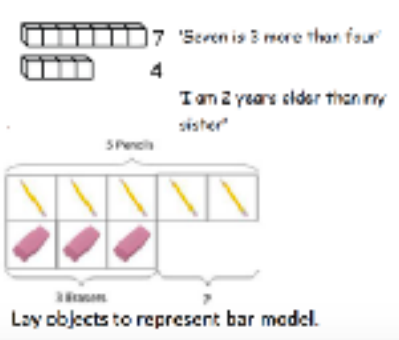
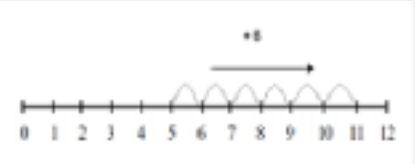
<p>Year 6 - add several numbers of increasing complexity</p> <p>Including money, measure and decimals with different numbers of digits after the decimal point</p>	<p>As year 5</p>	<p>As year 5</p>	<p>Insert zeros for place holders</p> <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="border: 1px solid black; padding: 5px;"> $\begin{array}{r} 81,059 \\ 3,668 \\ 15,301 \\ + 20,551 \\ \hline 120,579 \\ \small 1\ 1\ 1\ 1\ 1 \end{array}$ </div> <div style="border: 1px solid black; padding: 5px;"> $\begin{array}{r} 23.361 \\ 9.080 \\ 59.770 \\ + 1.300 \\ \hline 93.511 \\ \small 2\ 1\ 2\ 1\ 1 \end{array}$ </div> </div>
--	------------------	------------------	---



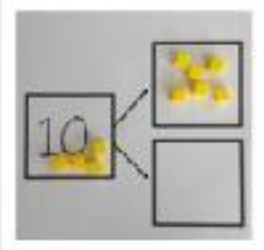
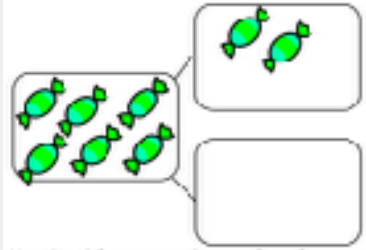
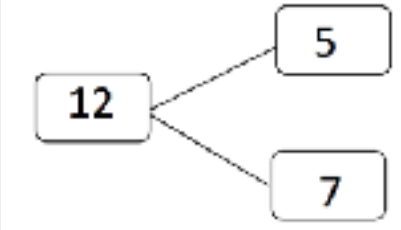

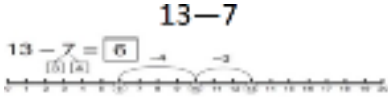



Subtraction – Year 1

Objective and strategy	Concrete	Pictorial	Abstract
<p>Taking away ones</p>	<p>Use physical objects, counters, cubes and others to show how objects can be taken away</p> 	<p>Cross out drawn objects to demonstrate what has been taken away</p> 	
<p>Counting back</p>	<p>Move objects away from the group counting backwards. Move the beads along the bead string as you count backwards.</p> 	<p>Count back in ones using a number line</p> 	<p>Put 13 in your head; count back 4. What number are you at?</p>

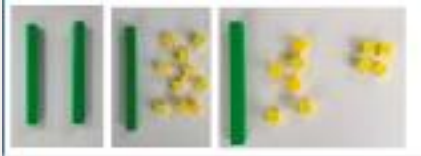


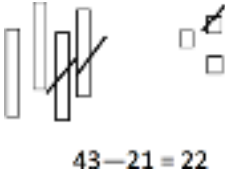


Find the difference	<p>Compare objects and amounts</p> 	<p>Count on using a number line to find the difference</p> 	<p>Hannah has 12 sweets and her sister has 5. How many more does Hannah have than her sister?</p>

Subtraction Year 1 cntd.	Concrete	Pictorial	Abstract
---------------------------------	-----------------	------------------	-----------------

<p>Represent and use number bonds and related subtraction facts within twenty</p> <p>Part whole model</p>	<p>Link to addition methods. Use part whole model to demonstrate the inverse ($10 - 6 = 4$)</p> 	<p>Use pictorial representations to show the part</p> 	<p>Move towards using numbers within the part whole model</p> 
<p>Make ten</p>	<p>Make 14 on the ten frame. Take 4 away to make ten and then take one more away so you have subtracted 5</p> <p>$14 - 9$</p> 	<p>Jump back 3 first and then another four. Use ten as the stopping point</p> <p>$13 - 7$</p> 	<p>16 - 8</p> <p>How many do we take off first to get to ten? How many do we have left to take off?</p>
<p>Introduce the bar model</p>	 <p>$5 - 2 = 3$</p>		 <p>$10 = 8 + 2$</p> <p>$10 = 2 + 8$</p> <p>$10 - 2 = 8$</p> <p>$10 - 8 = 2$</p>

Subtraction Year 2

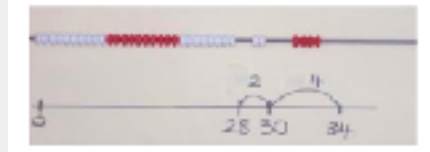
Objective and strategy	Concrete	Pictorial	Abstract
<p>Regroup a ten into 10 ones</p>	<p>Use a place value chart to demonstrate how to turn a ten into ten ones. Use the term: 'take and make.'</p> 		<p>$20 - 4 = 16$</p>
<p>Partitioning to subtract without regrouping</p> <p>Use 'friendly' numbers</p>	<p>Use dienes to demonstrate how to partition the number when subtracting without regrouping - $34 - 13 = 21$</p> 	<p>Children draw representations of dienes and cross off</p>  <p>$43 - 21 = 22$</p>	<p>$43 - 21 = 22$</p>



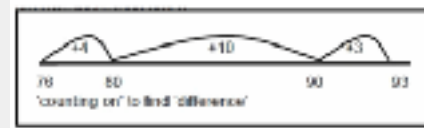
Make ten strategy

Progression should be crossing one ten, crossing more than one ten, crossing the hundreds.

Use a bead string to model counting to next ten and the rest



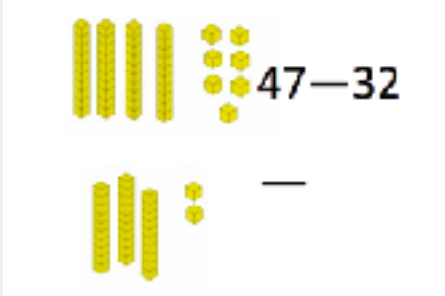
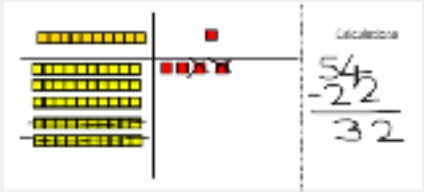
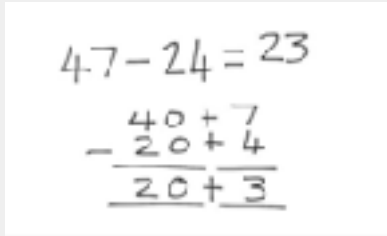

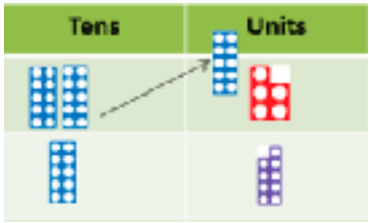
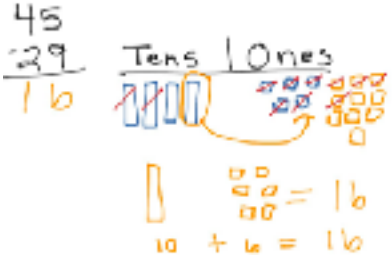


Use a number line to count on to next ten and then the rest



93 - 76 = 17

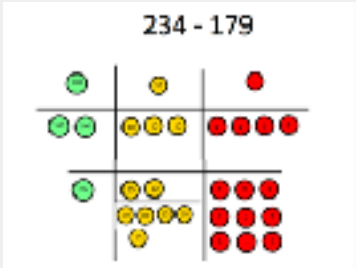
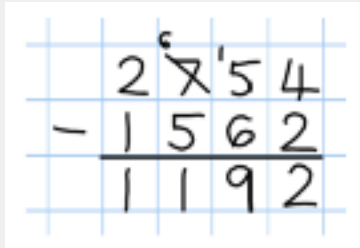
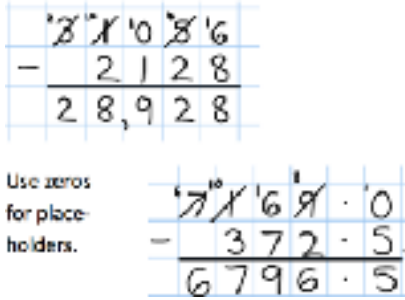


Subtraction Year 3

Objective and strategy	Concrete	Pictorial	Abstract
<p>Column subtraction without regrouping. Still using friendly numbers.</p>	<p>Use base ten or numicon to model</p> 	<p>Draw representations to support the children's understanding</p> 	<p>An intermediate step may be needed to lead to clear subtraction understanding</p>  
<p>Column subtraction with regrouping</p>	<p>Begin with base ten or numicon. Move to counters modelling the exchange of a ten into ten ones. Use the phrase 'take and make' for exchange</p> 	<p>Children may draw base ten or counters and cross off.</p> 	<p>Begin by partitioning into columns. Then move to formal method.</p>  



Subtraction Years 4-6

Objective and strategy	Concrete	Pictorial	Abstract
<p>Year 4 - subtracting tens and ones</p> <p>Subtract, eventually, with up to 4 digits</p> <p><i>Introduce decimal subtraction through the context of money</i></p>	<p>Model process of exchange using numicon, base ten and then move to counters.</p> 	<p>Children to draw counters and show their exchange (see year 3)</p>	<p>Use the phrase 'take and make' for the exchange</p> 
<p>Year 5 - Subtract with at least 4 digits including in the context of both money and measures</p> <p><i>Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal</i></p>	<p>As year 4</p>	<p>Children to draw counters and show their exchange (see year 3)</p>	 <p>Use zeros for place-holders.</p>



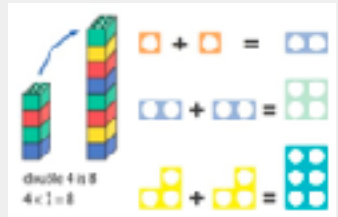
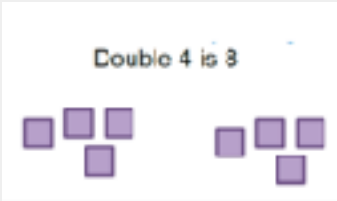
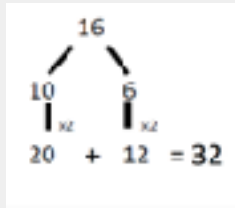

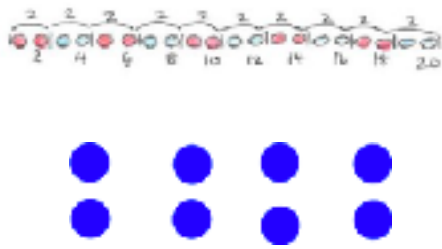
Year 6 - Subtract with increasingly large and complex numbers (also including decimal values)

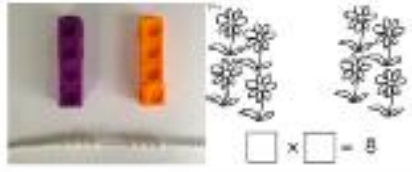
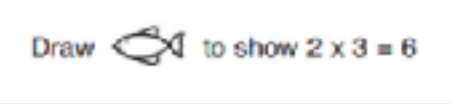
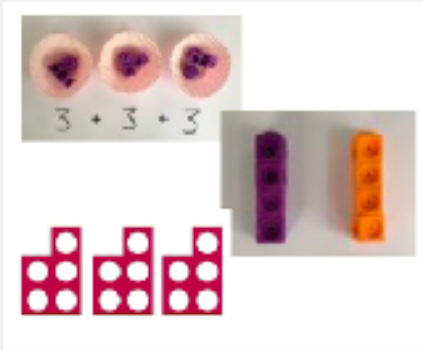
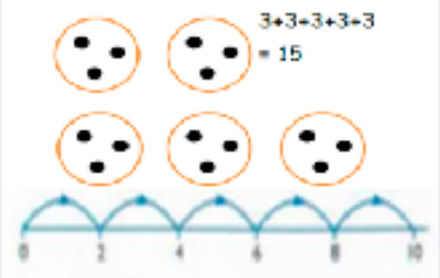

$$\begin{array}{r} \cancel{7} \cancel{8} \cancel{0} \cancel{6} \text{ } 9 \text{ } 9 \\ - \quad 8 \text{ } 9 \text{ } 9 \text{ } 4 \text{ } 9 \\ \hline 6 \text{ } 0 \text{ } 7 \text{ } 5 \text{ } 0 \end{array}$$

$$\begin{array}{r} \cancel{7} \cancel{0} \text{ } 5 \text{ } \cdot \text{ } \cancel{4} \text{ } 1 \text{ } 9 \text{ } \text{kg} \\ - \quad 3 \text{ } 6 \text{ } \cdot \text{ } 0 \text{ } 8 \text{ } \text{kg} \\ \hline 6 \text{ } 9 \text{ } \cdot \text{ } 3 \text{ } 3 \text{ } 9 \text{ } \text{kg} \end{array}$$



Multiplication – Year 1

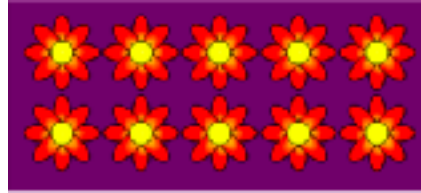
Objective and strategy	Concrete	Pictorial	Abstract
<p>Doubling</p>	<p>Use various practical activities (using manipulative) including cubes and numicon to demonstrate doubling</p> 	<p>Draw illustrations to demonstrate how to double numbers</p> 	<p>Partition a number before doubling each part and then recombining it back together</p> 
<p>Counting in multiples</p>	<p>Count the groups as children are counting (children may use their fingers as they are counting)</p> 	<p>Children make representations to show counting in multiples</p> 	<p>Aloud, count in multiples of a numbers Write sequences with multiples of numbers</p> <p style="text-align: center;">2, 4, 6, 8, 10</p> <p style="text-align: center;">5, 10, 15, 20, 25, 30</p>

<p>Making equal groups and counting the total</p>	<p>Use manipulative to create equal groups</p> 	<p>Draw and make representations</p> 	<p>$2 \times 4 = 8$</p>
<p>Multiplication Year 1 Continued</p>	<p>Concrete</p>	<p>Pictorial</p>	<p>Abstract</p>
<p>Repeated addition</p>	<p>Use different objects to add equal groups</p> 	<p>Use pictorial representations (including number lines) to solve problems</p> 	<p>Write addition sentences to describe both objects and pictures</p> 

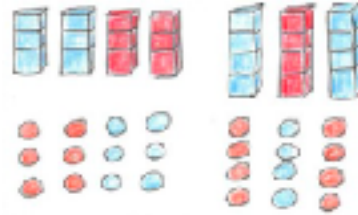


Show understanding of arrays

Use objects laid out in arrays to find the answers to 2 lots of 5, 3 lots of 2 and etc.



Draw representations of arrays to demonstrate understanding

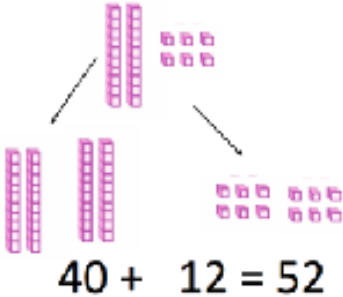
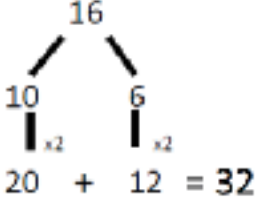
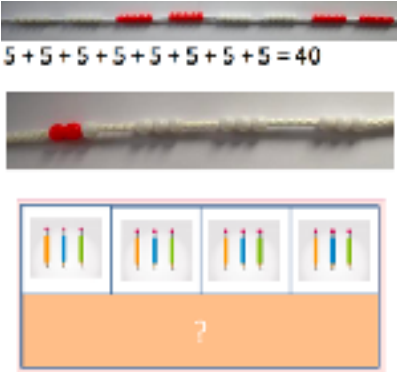
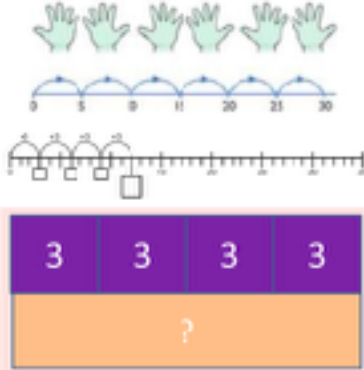


$$3 \times 2 = 6$$

$$2 \times 5 = 10$$



Multiplication – Year 2

Objective and strategy	Concrete	Pictorial	Abstract
<p>Doubling</p>	<p>Model doubling using dienes and counters</p>  <p style="text-align: center;">$40 + 12 = 52$</p>	<p>Draw pictures and representations to demonstrate how to double numbers</p>	<p>Partition a number and then double each part before recombining it back together</p>  <p style="text-align: center;">$20 + 12 = 32$</p>
<p>Counting in multiples of 2, 3, 4, 5, 10 from a startpoint of zero</p>	<p>Count the groups as children are counting (children may use their fingers as they are counting). Use bar models.</p> 	<p>Number lines, counting sticks and bar models should be used to show representations of counting in multiples</p> 	<p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers</p> <p>0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25, 30</p> <p style="text-align: center;">$4 \times 3 = \square$</p>

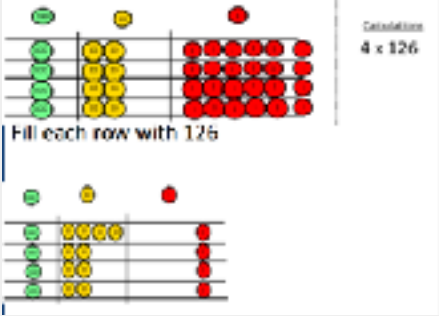
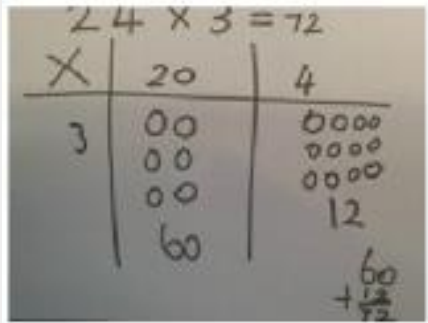
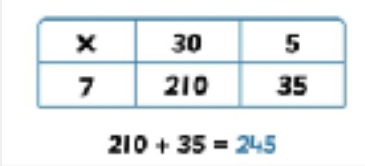


Multiplication – Year 3

Objective and strategy	Concrete	Pictorial	Abstract															
<p>Grid method</p>	<p>Show the links - with arrays - to first introduce the grid method</p> <p>Move onto base ten to move towards a more compact method.</p> <p>Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.</p> <p>Fill each row with 126</p> <p>Add up each column, starting with the ones making any exchanges needed</p> <p>Then you have your answer.</p>	<p>Children can represent their work with counters.</p> <p>They can draw the counters using colours to show different amounts or just use the circles in different columns to show their thinking (as below)</p> <p>Bar model are used to explore missing numbers</p>	<p>Start with multiplying by one digit numbers and showing the clear addition alongside the grid</p> <table border="1" style="margin: 10px auto; border-collapse: collapse;"> <tr> <td style="padding: 5px;">x</td> <td style="padding: 5px;">30</td> <td style="padding: 5px;">5</td> </tr> <tr> <td style="padding: 5px;">7</td> <td style="padding: 5px;">210</td> <td style="padding: 5px;">35</td> </tr> </table> <p style="text-align: center;">$210 + 35 = 245$</p> <p>Moving forward, multiply by a 2 digit number showing the different rows within the grid method.</p> <table border="1" style="margin: 10px auto; border-collapse: collapse;"> <tr> <td></td> <td style="padding: 5px;">10</td> <td style="padding: 5px;">8</td> </tr> <tr> <td style="padding: 5px;">10</td> <td style="padding: 5px;">100</td> <td style="padding: 5px;">80</td> </tr> <tr> <td style="padding: 5px;">3</td> <td style="padding: 5px;">30</td> <td style="padding: 5px;">24</td> </tr> </table>	x	30	5	7	210	35		10	8	10	100	80	3	30	24
x	30	5																
7	210	35																
	10	8																
10	100	80																
3	30	24																



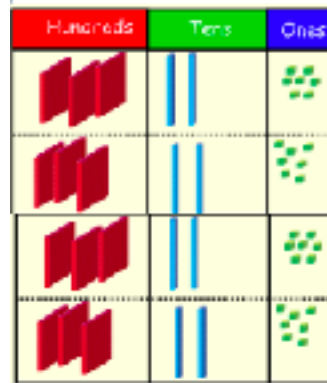
Multiplication – Year 4

Objective and strategy	Concrete	Pictorial	Abstract
<p>Grid method recap (do this for 2 digit multiplied by 1 digit)</p> <p>Move to multiplying 3 digit numbers by 1 digit numbers</p>	<p>Use place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows</p> 	<p>Children can represent their work with counters in a way that they understand.</p> 	<p>Start with multiplying by one digit numbers and showing the clear addition alongside the grid</p> 



Column multiplication

Children can continue to be supported by counters at this stage of multiplication. Begin with no regrouping



The grid method may be used to show how this relates to a formal written method



Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.

This may lead to a more compact method....

$$\begin{array}{r}
 327 \\
 \times 4 \\
 \hline
 1200 \\
 80 \\
 28 \\
 \hline
 1308
 \end{array}$$

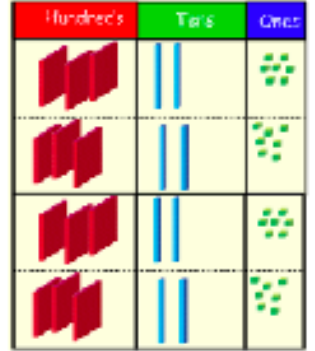
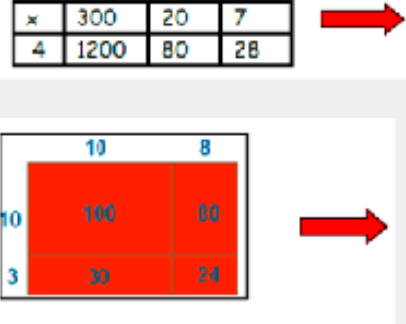
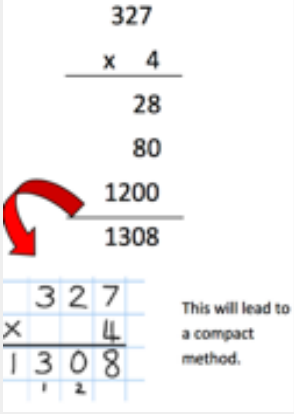
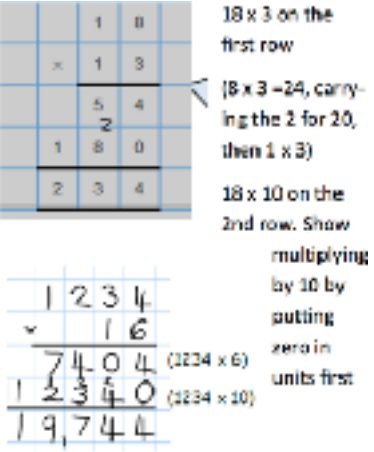


$$\begin{array}{r}
 327 \\
 \times 4 \\
 \hline
 1308
 \end{array}$$

This may lead to a compact method.

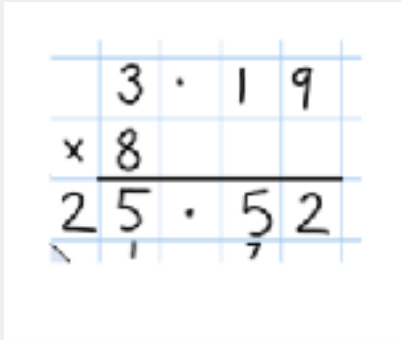


Multiplication – Year 5-6

Objective and strategy	Concrete	Pictorial	Abstract
<p>Column multiplication for 3 and 4 digits multiplied by 1 digit</p>	<p>Children can continue to be supported by counters at this stage of multiplication. Initially to be taught with no regrouping</p> 		 <p style="text-align: right;">This will lead to a compact method.</p>
<p>Column multiplication</p>	<p>Objects may still be used with the corresponding long multiplication modelled alongside</p>	<p>(Continue to use bar modelling to support problem solving)</p>	

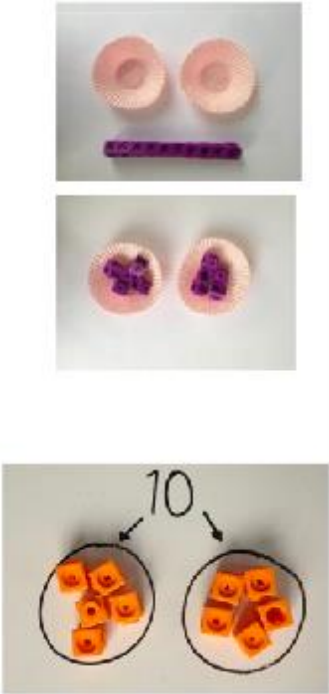
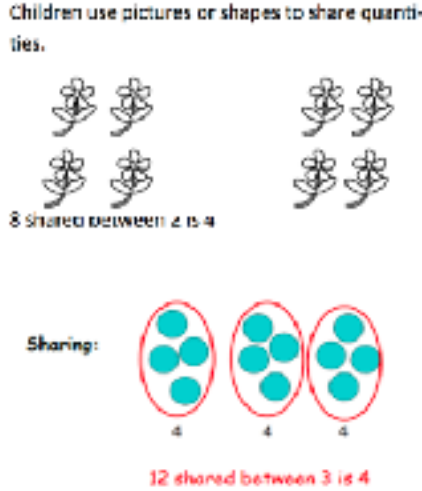


Multiplication – Year 6

Objective and strategy	Concrete	Pictorial	Abstract
Multiplying decimals (up to 2 decimal places) by a single digit			Remind the children that the single digit belongs in the ones column. Use the decimal points in the question and answer 

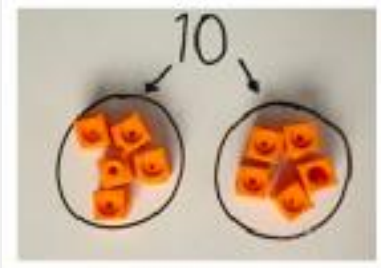
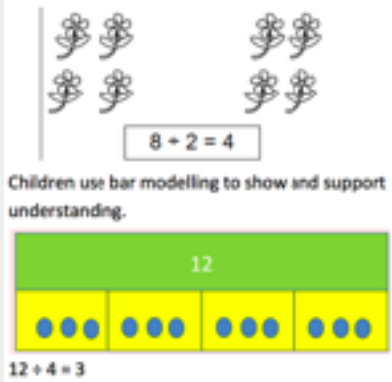
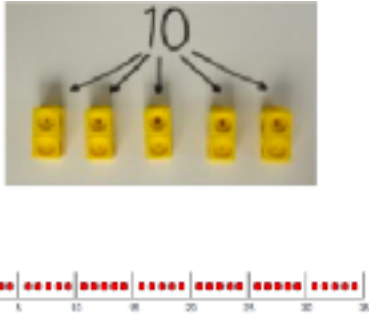
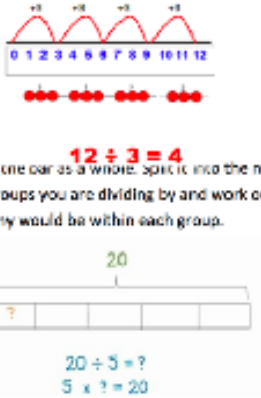


Division – Year 1

Objective and strategy	Concrete	Pictorial	Abstract
<p>Division as sharing</p>	<p>I have ten cubes - can you share them equally in 2 groups?</p> 	<p>Children use pictures (or shapes) to share quantities</p> <p>Children use pictures or shapes to share quantities.</p> 	<p>12 shared between 3 is 4</p>





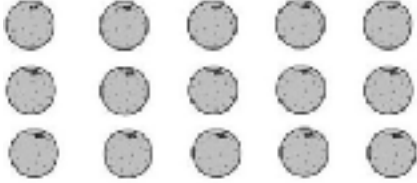


Division – Year 2

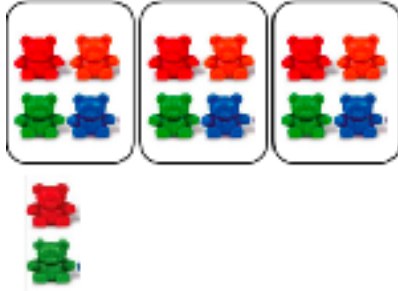


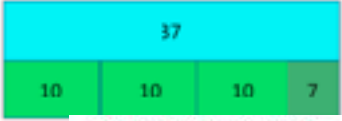
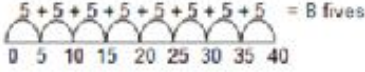
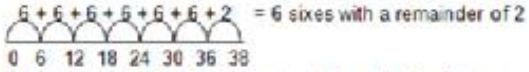
Objective and strategy	Concrete	Pictorial	Abstract
<p>Division as sharing</p>	<p>I have ten cubes. Can you share them equally between 2 groups</p> 	<p>Children use pictures or shapes to share quantities</p>  <p>Children use bar modelling to show and support understanding.</p>	<p>12 divided by 3 equals 4</p>
<p>Division as grouping</p>	<p>Divide quantities into equal groups</p> <p>Use cubes, counters or objects to add the children's understanding</p> 	<p>Use number lines for grouping</p>  <p>Think of one bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.</p>	<p>28 divided by 7 equals 4</p> <p>Divide 28 into 7 groups. How many are in each group?</p>



Division – Year 3

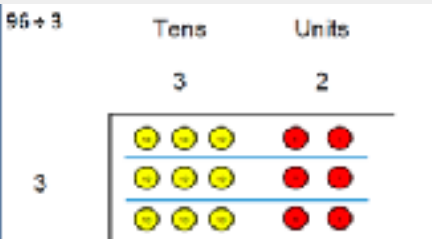


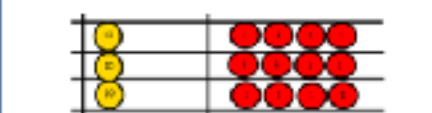
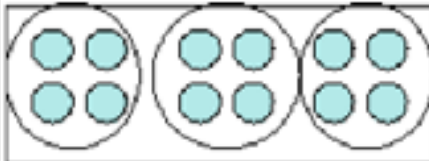
Objective and strategy	Concrete	Pictorial	Abstract
<p>Division as grouping</p>	<p>Use cubes, counters or other objects to aid the children’s understanding</p>  <p>24 divided into groups of 5 = 4</p> $96 \div 3 = 32$ 	<p>Continue to use bar modelling to aid solving division related problems</p>  $20 \div 5 = ?$ $5 \times ? = 20$	<p>How many groups of 6 in 24?</p> <p>24 divided by 6 equals 4</p>
<p>Division with arrays</p>	<p>Link division to multiplication by creating an array and thinking about the number sentences that can be created.</p>  <p>Eg $15 \div 3 = 5$ $5 \times 3 = 15$ $15 \div 5 = 3$ $3 \times 5 = 15$</p>	<p>Draw an array, using lines to split the array into groups to make multiplication and division sentences</p> 	<p>Find the inverse of multiplication and division sentences by creating eight linking number sentences</p> $7 \times 4 = 28$ $4 \times 7 = 28$ $28 \div 7 = 4$ $28 \div 4 = 7$ $28 = 7 \times 4$ $28 = 4 \times 7$ $4 = 28 \div 7$ $7 = 28 \div 4$

Division – Year 4

Objective and strategy	Concrete	Pictorial	Abstract
<p>Division with remainders</p>	<p>14 divided by 3</p> <p>Divide objects between groups and see how much is left over.</p> 	<p>Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder</p>  <p>Draw dots and group them to divide an amount and clearly show a remainder.</p>  <p>Use bar models to show division with remainders.</p>  <p>Example without remainder: $40 \div 5$ Ask "How many 5s in 40?"</p>  <p>Example with remainder: $38 \div 6$</p>  <p>For larger numbers, when it becomes inefficient to count in single multiples, bigger jumps can be recorded using known facts.</p>	<p>Complete written divisions and show the remainder through the use of r</p> $29 \div 8 = 3 \text{ REMAINDER } 5$ <p style="text-align: center;"> ↑ ↑ ↑ ↑ childend divisor quotient remainder </p>



Division – Year 4-6

Objective and strategy	Concrete	Pictorial	Abstract				
<p>Divide, at least, 3 digit numbers by 1 digit</p> <p>Work towards short division</p>	<div style="text-align: center;"> $66 \div 3$ <table style="display: inline-table; border-collapse: collapse;"> <tr> <td style="padding: 0 10px;">Tens</td> <td style="padding: 0 10px;">Units</td> </tr> <tr> <td style="text-align: center;">3</td> <td style="text-align: center;">2</td> </tr> </table> </div>  <p>Use place value counters to divide using the bus stop method alongside</p>  <p>$42 \div 3 =$</p> <p>Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.</p>  <p>We exchange this ten for ten ones and then share the ones equally among the groups.</p>  <p>We look how much in 1 group so the answer is 14.</p>	Tens	Units	3	2	<p>Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.</p> <p>Encourage them to move towards counting in multiples to divide more efficiently</p> 	<p>Begin by teaching divisions that divide equally with no remainder</p> <div style="text-align: center;"> $\begin{array}{r} 218 \\ 4 \overline{) 872} \end{array}$ </div> <p>Move onto divisions with a remainder.</p> <div style="text-align: center;"> $\begin{array}{r} 86 \text{ r } 2 \\ 5 \overline{) 432} \end{array}$ </div> <p>Finally move into decimal places to divide the total accurately.</p> <div style="text-align: center;"> $\begin{array}{r} 14.6 \\ 35 \overline{) 511.0} \end{array}$ </div> <div style="text-align: center;"> $\begin{array}{r} 0663 \text{ r } 5 \\ 8 \overline{) 5309} \end{array}$ </div>
Tens	Units						
3	2						



Long division

Step 1 - a remainder in the ones

$$\begin{array}{r} \text{h t o} \\ 041 \text{ R}1 \\ \hline 4 \overline{) 165} \end{array}$$

4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).

4 goes into 16 four times.

4 goes into 5 once, leaving a remainder of 1.

$$\begin{array}{r} \text{th h t o} \\ 0400 \text{ R}7 \\ \hline 8 \overline{) 3207} \end{array}$$

8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200).

8 goes into 32 four times ($3,200 \div 8 = 400$)

8 goes into 0 zero times (tens).

8 goes into 7 zero times, and leaves a remainder of 7.



Long division

Step 1 continued....

$$\begin{array}{r} \text{h t o} \\ 061 \\ 4 \overline{) 247} \\ \underline{-4} \\ 3 \end{array}$$

When dividing the ones, 4 goes into 7 one time. Multiply $1 \times 4 = 4$, write that four under the 7, and subtract. This finds us the remainder of 3.

Check: $4 \times 61 + 3 = 247$

$$\begin{array}{r} \text{th h t o} \\ 0402 \\ 4 \overline{) 1609} \\ \underline{-8} \\ 1 \end{array}$$

When dividing the ones, 4 goes into 9 two times. Multiply $2 \times 4 = 8$, write that eight under the 9, and subtract. This finds us the remainder of 1.

Check: $4 \times 402 + 1 = 1,609$



Long division

Step 2 - a remainder in the tens

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{t o} \\ 2 \overline{)58} \end{array}$ <p>Two goes into 5 two times, or 5 tens + 2 = 2 whole tens -- but there is a remainder!</p>	$\begin{array}{r} \text{t o} \\ 2 \overline{)58} \\ -4 \\ \hline 1 \end{array}$ <p>To find it, multiply $2 \times 2 = 4$, write that 4 under the five, and subtract to find the remainder of 1 ten.</p>	$\begin{array}{r} \text{t o} \\ 29 \\ 2 \overline{)58} \\ -4 \\ \hline 18 \end{array}$ <p>Next, drop down the 8 of the ones next to the leftover 1 ten. You combine the remainder ten with 8 ones, and get 18.</p>
$\begin{array}{r} \text{t o} \\ 29 \\ 2 \overline{)58} \\ -4 \\ \hline 18 \end{array}$ <p>Divide 2 into 18. Place 9 into the quotient.</p>	$\begin{array}{r} \text{t o} \\ 29 \\ 2 \overline{)58} \\ -4 \\ \hline 18 \\ -18 \\ \hline 0 \end{array}$ <p>Multiply $9 \times 2 = 18$, write that 18 under the 18, and subtract.</p>	$\begin{array}{r} \text{t o} \\ 29 \\ 2 \overline{)58} \\ -4 \\ \hline 18 \\ -18 \\ \hline 0 \end{array}$ <p>The division is over since there are no more digits in the dividend. The quotient is 29.</p>



Long division

Step 2 continued (a remainder in any of the place

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{h t o} \\ 1 \\ 2 \overline{) 278} \end{array}$ <p>Two goes into 2 one time, or 2 hundreds + 2 = 1 hundred.</p>	$\begin{array}{r} \text{h t o} \\ 1 \\ 2 \overline{) 278} \\ \underline{-2} \\ 0 \end{array}$ <p>Multiply $1 \times 2 = 2$, write that 2 under the two, and subtract to find the remainder of zero.</p>	$\begin{array}{r} \text{h t o} \\ 18 \\ 2 \overline{) 278} \\ \underline{-2} \\ 07 \end{array}$ <p>Next, drop down the 7 of the tens next to the zero.</p>
Divide.	Multiply & subtract.	Drop down the next digit.
$\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{) 278} \\ \underline{-2} \\ 07 \end{array}$ <p>Divide 2 into 7. Place 3 into the quotient.</p>	$\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{) 278} \\ \underline{-2} \\ 07 \\ \underline{-6} \\ 1 \end{array}$ <p>Multiply $3 \times 2 = 6$, write that 6 under the 7, and subtract to find the remainder of 1 ten.</p>	$\begin{array}{r} \text{h t o} \\ 138 \\ 2 \overline{) 278} \\ \underline{-2} \\ 07 \\ \underline{-6} \\ 18 \end{array}$ <p>Next, drop down the 8 of the ones next to the 1 leftover ten.</p>
1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{) 278} \\ \underline{-2} \\ 07 \\ \underline{-6} \\ 18 \end{array}$ <p>Divide 2 into 18. Place 9 into the quotient.</p>	$\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{) 278} \\ \underline{-2} \\ 07 \\ \underline{-6} \\ 18 \\ \underline{-18} \\ 0 \end{array}$ <p>Multiply $9 \times 2 = 18$, write that 18 under the 18, and subtract to find the remainder of zero.</p>	$\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{) 278} \\ \underline{-2} \\ 07 \\ \underline{-6} \\ 18 \\ \underline{-18} \\ 0 \end{array}$ <p>There are no more digits to drop down. The quotient is 139.</p>



